Metamorphosis of Freeform Curves and Surfaces

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Abstract

Metamorphosis between two freeform B-spline curves is considered and several approaches to control the process are discussed. Starting with simple convex combination, we examine two other approaches, one based on multiresolution decomposition of freeform curves and the other based on edge cutting of the control polygon of curves. The later is improved by introducing a correspondence test for simple metamorphosis relation. Finally, we consider the possibility of extending these algorithms to surfaces.

Keywords: Morphing, Freeform, Correspondence

1 Introduction

Continuous shape deformation or metamorphosis has captured its place as a major tool in animation. The problem has been investigated in the context of two dimensional images [1, 18], curves and surfaces [8, 13, 14, 15, 16], and even voxel based volumetric representations [9, 11]. Several techniques were developed in recent years to compute the metamorphosis of two dimensional univariate functions, the problem we address herein.

Probably the easiest approach to the metamorphosis of two piecewise linear polylines, $\mathcal{P}_1$ and $\mathcal{P}_2$, is to establish the correspondence between the vertices of $\mathcal{P}_1$, V_1^i, $0 \leq i \leq l_1$, and the vertices of $\mathcal{P}_2$, V_2^i, $0 \leq i \leq l_2$, for example by introducing new vertices. Then, the metamorphosis is computed using a convex combination of the corresponding vertices as $V = tV_1^i + (1-t)V_2^i$, $0 \leq i \leq l$.

Unfortunately, this somewhat naive approach, can lead to artifacts that disqualify the result from been considered an acceptable metamorphosis. Intermediate shapes during the metamorphosis can vanish and degenerate into a point (Consider the metamorphosis of the line $\mathcal{P}_1 = ((-1,-1),(-1,1))$ and the line $\mathcal{P}_2 = ((1,1),(1,-1))$ at $t = 0.5$) or self intersect even if

neither one of $\mathcal{P}_1$ or $\mathcal{P}_2$ self intersect. Hence, most of the research of metamorphosis of two dimensional univariate functions has been concentrated on the elimination of these artifacts from the computed metamorphosis.

In [14, 15], piecewise linear polylines are metamorphed by deriving a heuristic algorithm that takes into account angles between adjacent edges as well as the lengths of the edges. In [16], the geometry of both polylines is preprocessed into an intermediate representation called a skeleton which contains topological information on the shape and simplifies the process of correspondence establishment between the two shapes. In [8], multiresolution decompositions of the two (closed) polygons is precomputed and the metamorphosis process is computed between the different resolution representations of the two curves only to be combined into the final result metamorphosis, in the end.

All the above work exploits piecewise linear polylines as the representation of choice and it is difficult to extend this work to freeform curves. Moreover, none of the above algorithms guarantee a self intersecting free metamorphosis. Finally, some manual intervention, in different levels, is required in all the above metamorphosis techniques for piecewise linear polyline. For example, the correspondence between vertices in $\mathcal{P}_1$ and $\mathcal{P}_2$ is typically established by manually correlating a small subset of vertices in both polylines. Nevertheless, in this paper, we question the feasibility of an automatic metamorphosis between two freeform curves directly. In Section 2, we are introduced to the metamorphosis of freeform curves through a simplistic approach. In Sections 3 and Section 4, we examine two more sophisticated algorithms. One (Section 3) examines the possibility of extending the multiresolution approach suggested in [8] to freeform shapes and the other (Section 4) draws some ideas from corner cutting smoothing techniques [10, 17].

All the examples and figures in this paper were created using the IRIT [12] solid modeller that is being developed at the Technion.

2   Metamorphosis using Simple Convex Combination

Let $C_1(u) = \sum_{i=0}^{l_1} P_i^1 B_i^{n_1}(u)$ and $C_2(u) = \sum_{i=0}^{l_2} P_i^2 B_i^{n_2}(u)$ be two arbitrary B-spline curves of orders n_1 and n_2 respectively. $C_1(u)$ and $C_2(u)$ will not share the same order, nor would they possess the same continuity (knot vector), in general. However, using degree raising [4] and refinement [2, 3], one can elevate the two curves to a common function space, in which both share the same order $n = \max(n_1, n_2)$ and continuity. Let $\hat{C}_i(u)$ represent $C_i(u)$ in the common function space. Obviously, $\hat{C}_1(u)$ and $\hat{C}_2(u)$ share the same number of control points l.

The control polygons of a B-spline curve resembles the shape of its curve. In fact, the control polygon, after proper refinement, is sometimes employed as an approximation to the curve, for display purposes. In order to form a continuous metamorphosis between $C_1(u)$ and $C_2(u)$ at time t, $C(u, t)$, one can compute the appropriate convex combination of the points of their control polygons,

$$
\begin{aligned}
C(u, t) &= (1 - t)\hat{C}_1(t) + t\hat{C}_2(t) \\
&= (1 - t)\sum_{i=0}^{l} P_i^1 B_i^n(u) + t\sum_{i=0}^{l} P_i^2 B_i^n(u) \\
&= \sum_{i=0}^{l}(1 - t)P_i^1 B_i^n(u) + \sum_{i=0}^{l} tP_i^2 B_i^n(u) \\
&= \sum_{i=0}^{l}\left((1 - t)P_i^1 + tP_i^2\right) B_i^n(u),
\end{aligned}
\tag{1}
$$

and hence, the convex combination of the corresponding control points is the same as the convex combination of the two curves. t is constraint to be between zero and one for an interpolation.

Figure 1 shows one example exploiting this approach. Unfortunately, this somewhat naive approach frequently fails due to inabilities in satisfying fundamental requirements that are necessary for a metamorphosis to be considered acceptable. Two non self intersecting curves, $C_i(t)$, $i = 1, 2$ might be metamorphed using Equation (1) to a degenerated point or to a self intersecting intermediate curve. See, for example, Figure 2.

Eliminating the self intersection in intermediate steps is a necessary requirement in creating a pleasing metamorphosis. Unfortunately, it is a difficult problem that partial solutions are known only for metamorphosis between piecewise linear polylines [8, 14, 15, 16]. In the next two
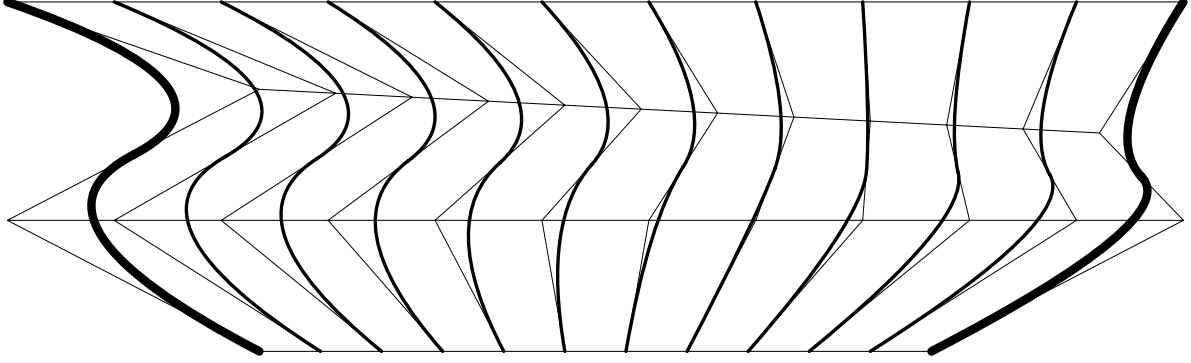
Figure 1: A metamorphosis between two B-spline curves computed using a convex combinations of the two curves, sharing a common function space.
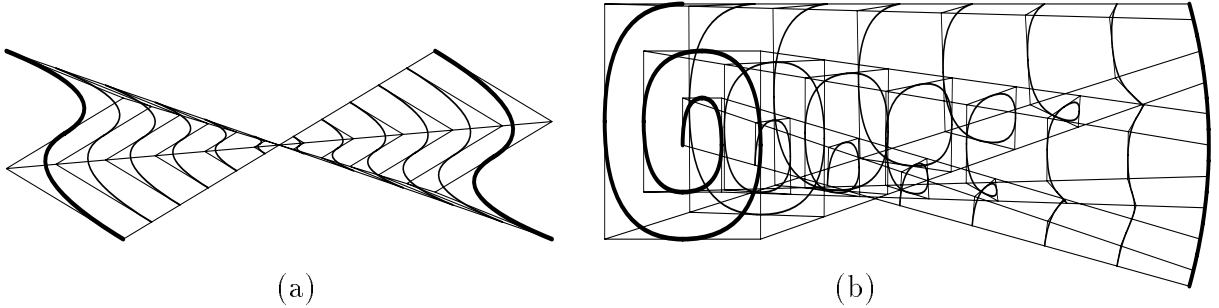

(a) (b)

Figure 2: A metamorphosis between two B-spline curves computed using a convex combinations of the two curves can lead to vanishing edges (a) or self intersection (b) in the intermediate representations.

sections, Section 3 and Section 4, we examine two attempts at the creation of self intersection free metamorphosis of freeform B-spline curves.

3 Metamorphosis using Multiresolution Decomposition

Multiresolution decomposition methods of freeform curves and surfaces where investigated in the context of curve manipulation [6, 7] and data reduction [5]. Recently, multiresolution decomposition methods were suggested as tools for morphing of piecewise linear polylines [8]. Herein, we examine the possibility of extending [8] to freeform B-spline curves.

Let $C(u)$ be a B-spline curve, $C(u) \in W^0$. W^0 is the function space defined over τ^0, the knot vector of $C(u)$. Define a set of lower resolution function spaces W^j, $0 < j \leq k$, such that $W^{j+1} \subset W^j$. By computing the least squares projection of $C(u)$ onto W^j as $C^j(u)$, one can

define,

$$C(u) = C^k(u) + \sum_{j=k}^{1} D^j(u), \tag{2}$$

where $D^j(u) = C^{j-1}(u) - C^j(u)$. The set $\{D^1(u), \cdots, D^k(u), C^k(u)\}$ is called the multiresolution decomposition of $C(u)$. See [6] for more.

$C^k(u) \in W^k$, the lowest resolution curve, conveys the generic shape of the curve. The vector field $D^1(u) \in W^0$ expresses the finest details of $C(u)$. It is plausible that the metamorphosis between the two curves can be made more pleasing by controlling the metamorphosis in different resolutions, an argument suggested in [8].

Let $C_1(u)$ and $C_2(u)$ be two B-spline curves that one would like to compute an inbetweening metamorphosis for. Assume $C_i(u)$, $i = 1, 2$ share the same function space. Then, following [6], the vector field curves, $D_i^j(u)$, resulting from the multiresolution decompositions of $C_i(u)$ share the same function spaces. As in Section 2, one can attempt and apply a convex combination to each of the pairs of the different corresponding vector fields $D_i^j(u)$, $i = 1, 2$, $0 < j < k$ as well as $C_i^k(u)$, $i = 1, 2$, and algebraically some the results, following Equation (2). However, due to the linearity of the spaces this would lead to the exact same result as in Section 2, namely, convex combination of $C_1(u)$ and $C_2(u)$ as in Equation (1). In [8], an additional approach, instead of exploiting a convex combination of two (control) points or vectors, is suggested. One could consider vertex $V_1^i \in \mathcal{P}_1$ as a vector and rotate it to $V_2^i \in \mathcal{P}_2$, while preserving the sizes of the vectors. We can immediately apply this approach herein and rotate every control point in the vector field of $D_1^j(u)$ to the corresponding control point of vector field $D_2^j(u)$ while appropriately compensating for the size of the vector.

While this approach does not guarantee self intersection free metamorphosis, one could hope for a more pleasing deformation. Figure 3 shows several examples of metamorphosis using multiresolution decomposition. The results are unsatisfactory and self intersections can become even more evident than in the convex combination. Moreover, little intuition can be found in the intermediate shapes themselves that can assume unexpected forms. Unlike piecewise linear polylines, curves have another degree of freedom which is their parametrization. It is probably the case that this technique would be greatly improved if proper correspondence between the
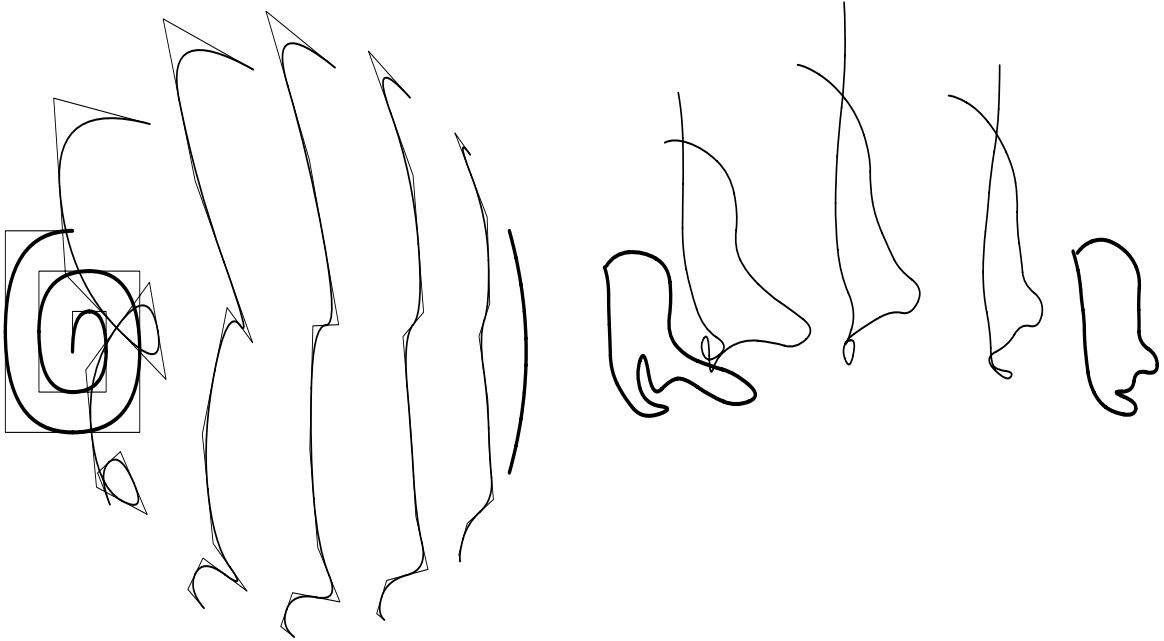
Figure 3: A metamorphosis between two B-spline curves computed using multiresolution decomposition produces non intuitive intermediate shapes and can lead to a self intersection in the intermediate representations.

two curves can be established, taking into account the different parametrization of the two curves.

4 Metamorphosis using Edge Cutting

Consider a curve with highly curved regions as well as almost zero curvature domains. Intuition suggests that the highly curved regions are more likely to self intersect during the metamorphosis stage than almost zero curvature domains. Hence, it is plausible to attempt and reduce the curvature of the curve at the highly curved regions first. Extending the corner cutting methodology [10], we proposed an *edge cutting* method that metamorphs an arbitrary open B-spline curve to a straight line, favoring the metamorphosis of high curvature regions first. Figure 4 shows several steps of an example of edge cutting a freeform B-spline into a line. While this approach does not guarantee a self intersection free metamorphosis, it does produce superior results in many instances, compared to both the naive convex combination of control points and the multiresolution decomposition approaches.

Let $C(u) = \sum_{i=0}^{l} P_i B_i^n(u)$ be a B-spline curve of order n and l control points. Consider three consecutive control points P_j, $j = i, i+1, i+2$ (See Figure 5). In every step of the

Figure 4: A metamorphosis of a B-spline curve to an almost straight curve using edge cutting, without self intersection. See also Figure 3.

metamorphosis process, we update control point P_{i+1} of $C(u)$ to be,

$$P_{i+1} \Leftarrow tP_{i+1} + (1-t)\frac{P_i + P_{i+2}}{2}, \tag{3}$$

where $0 \leq t \leq 1$ provides the control over the edge cutting speed that is typically close to one. Equation (3) is applied to all the interior control points P_{i+1}, $0 \leq i \leq l-2$. The two end points of the curve are hence left unchanged.

Lemma 1 *The edge cutting process defined using Equation (3) over an arbitrary open B-spline curve and applied to all interior control points converges to line $\overline{P_0 P_l}$.*
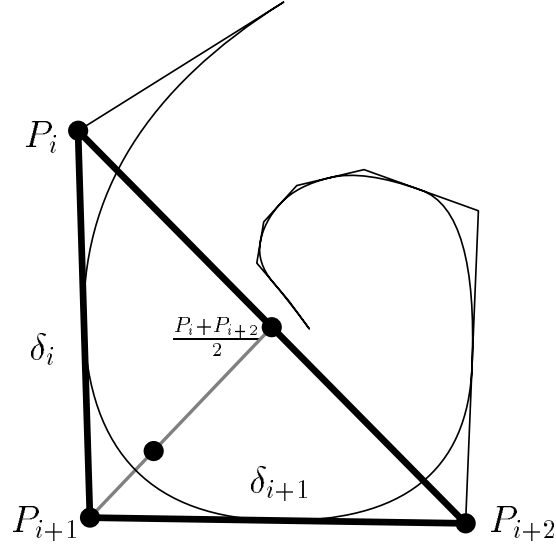
Figure 5: The control polygon of $C(u)$ is updated in every iteration of the edge cutting process, by letting every interior control points P_{i+1} to be $P_{i+1} \Leftarrow tP_{i+1} + (1-t)\frac{P_i + P_{i+2}}{2}$.

Proof: Clearly, a curve in which all its control points are collinear and equally spaced is a fixed point of the transformation defined in Equation (3).

On the other hand, transformation (3) always reduces the total length of the control polygon, provided triangle $P_i P_{i+1} P_{i+2}$ is non degenerated (i.e. not collinear). Specifically, the accumulated length of $\overline{P_i P_{i+1}} + \overline{P_{i+1} P_{i+2}}$ is always smaller after applying Equation (3). Therefore, successive iteration of Equation (3) must reduce the total length of the control polygon to a minimum which is the line $\overline{P_0 P_l}$, a line that we know is a fix point for Equation (3).

∎

Given two arbitrary open B-spline curves, $C_1(u)$ and $C_2(u)$, one can compute their edge cutting to lines $L_1(u)$ and $L_2(u)$. Let $L(u) = (L_1(u) + L_2(u))/2$. One can transform $L_i(u)$ to $L(u)$ using translation, scaling and rotation. Denote the transform from $L_i(u)$ to $L(u)$ by $\mathcal{R}_i$. The entire metamorphosis will then be composed of first transforming $C_1(u)$ to $L_1(u)$ while continuously and incrementally applying $\mathcal{R}_1$ so it ends up at $L(u)$ instead of $L_1(u)$. Then, the inverse transformation from line $L_2(u)$ to $C_2(u)$ will be computed, again starting from line $L(u)$ by decrementally applying $\mathcal{R}_2^{-1}$ to start at $L(u)$ and terminate at $C_2(u)$.

The edge cutting process can create intermediate control polygons in which one (or more) of

the edges vanishes. In order to prevent this from happening, the following heuristic is exploited. Let $\delta_i = \|P_{i+1} - P_i\|$ and let $\Delta = \min_i \delta_i$, $0 \le i \le l - 1$. We then redefine Equation (3) to be,

$$
\begin{aligned}
t_i &\Leftarrow t\left(\frac{\Delta}{\delta_{i+1}}\right)^2, \\
t_{i+2} &\Leftarrow t\left(\frac{\Delta}{\delta_i}\right)^2, \\
t_{i+1} &\Leftarrow 1 - t_i - t_{i+2}, \\
P_{i+1} &\Leftarrow t_i P_i + t_{i+1} P_{i+1} + t_{i+2} P_{i+2}.
\end{aligned}
\tag{4}
$$

The modified edge cutting algorithm favors the shortening of the longer edge out of the two edges $\overline{P_i P_{i+1}}$ and $\overline{P_{i+1} P_{i+2}}$ while computing the new location for P_{i+1}. At the same time, Equation (4) favors the decrease of small edges and hence the reduction of curvature near shorter edges over longer ones throughout the control polygon. This last heuristic is driven from the fact that short edges probably convey information in a higher resolution and therefore create high curvature regions in the curve. Figure 4 shows one example of curve's metamorphosis exploiting this approach. This metamorphosis has no self intersection, that will occur when both the naive and the multiresolution methods are applied to this problem.

Clearly, this approach suffers from a major drawback. We must produce an intermediate straight line half way through the metamorphosis process, no matter how close $C_1(u)$ and $C_2(u)$ are. Figure 6 shows two almost identical curves that are edge cut metamorphed through a straight line.

Fortunately, we can do better. Let $T_i(u) = \frac{dC_i(u)}{du}$. Define,

$$
\mathcal{T}(u) = \langle T_1(u), T_2(u) \rangle,
\tag{5}
$$

where $\langle \cdot, \cdot \rangle$ denotes the inner product. We say that $C_1(u)$ has a *simple metamorphosis relation* to $C_2(u)$ if $\mathcal{T}(u) > 0$, $\forall u$. If $\mathcal{T}(u) > 0$, $\forall u$, then one can, in general, form a non self intersecting ruled surface $S(u,v)$ between $C_1(u)$ and $C_2(u)$ while v is the metamorphosis parameter. The ruling direction, v, describes the convex combination metamorphosis from $C_1(u)$ to $C_2(u)$.

Lemma 2 *If $C_i(u)$, $i = 1, 2$ satisfy a simple metamorphosis relation, that is*

$$
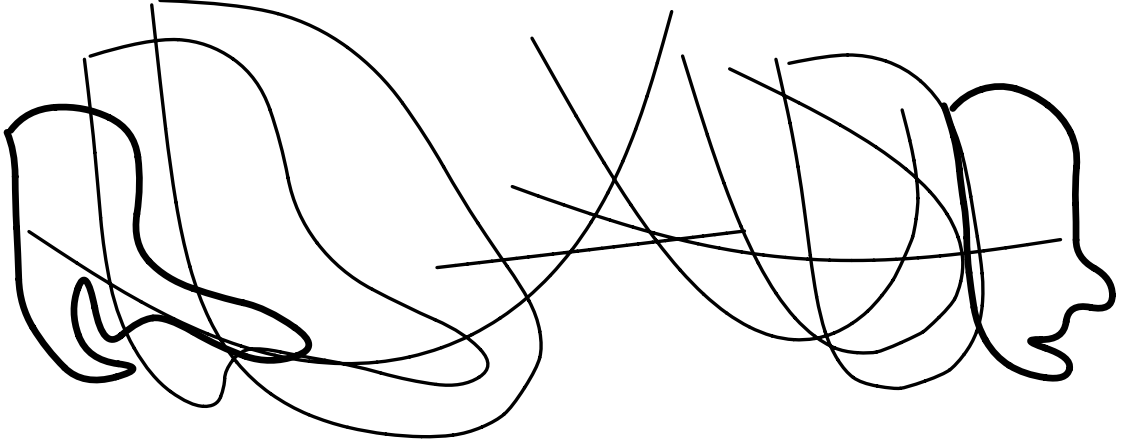\mathcal{T}(u) = \langle T_1(u), T_2(u) \rangle > 0, \ \forall u,
$$

Figure 6: A metamorphosis of between two similar face figures using edge cutting approach. Although we prevent from self intersection, we do generate a non intuitive metamorphosis by going through an intermediate straight line. Compare with Figure 3.

then $C(u) = tC_1(u) + (1 - t)C_2(u)$, $0 \leq t \leq 1$ is a regular curve.

Proof: $\mathcal{T}(u) > 0$ and hence $\|T_i(u)\| > 0$, $i = 1, 2$ or $C_i(u)$, $i = 1, 2$ are regular curves. $\frac{dC(u)}{du} = tT_1(u) + (1 - t)T_2(u)$ and is a linear combination of two vector fields such that the angle between $T_1(u)$ and $T_2(u)$ satisfies $\angle T_1(u)T_2(u) < 90^o$. Thus, the result of the linear combination of $T_1(u)$ and $T_2(u)$ is never zero.

∎

Although the guaranteed regularity of the intermediate curves is a nice property, the simple metamorphosis relation does prevent from self intersection of the intermediate curves only in a local neighborhood, contrary to what one might expect. Assume $C_i(u)$, $i = 1, 2$ satisfy a simple metamorphosis relation, that is $\mathcal{T}(u) = \langle T_1(u), T_2(u) \rangle > 0$, $\forall u$. $C(u) = tC_1(u) + (1 - t)C_2(u)$, $0 \leq t \leq 1$ can self intersect as is demonstrated in two examples in Figure 7. In Figure 7 (b), $C_1(u)$ intersects neither itself nor $C_2(u)$ and vice versa.

In the light of the inability to exactly predict self intersection, we found that using $\mathcal{T}(u)$ one can significantly alleviate the need to edge cut metamorph through a line, creating a more pleasing metamorphosis. This, while preventing self intersection in the majority of cases. Employing Equation (5), one can establish simple metamorphosis relation from the intermediate straight line representation $L(u)$ out to both $C_1(u)$ and $C_2(u)$. The metamorphosis domain
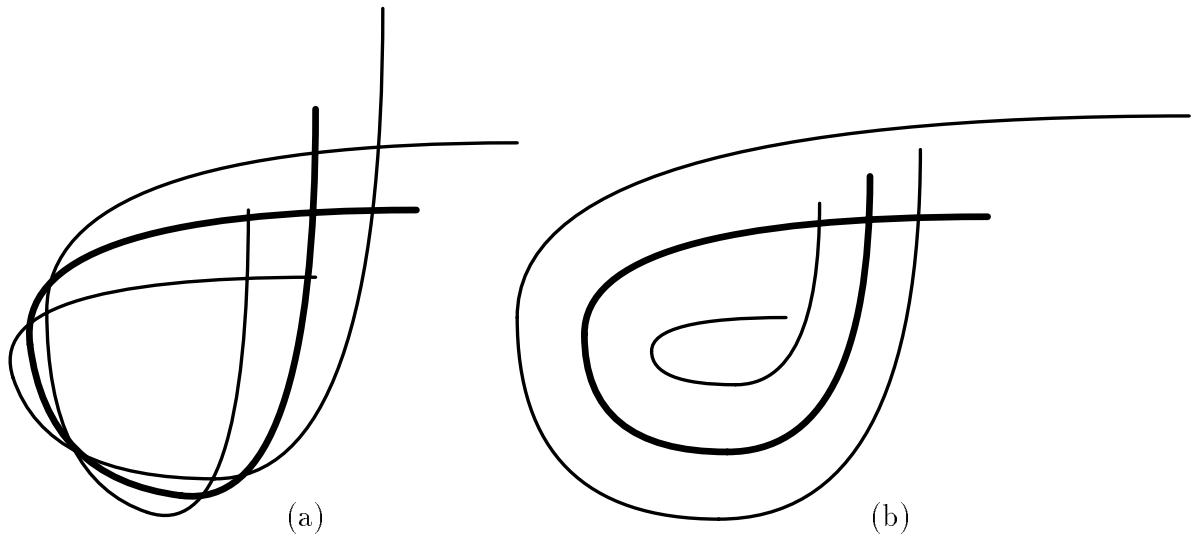
Figure 7: Two examples of intermediate curves $C(u) = tC_1(u) + (1 - t)C_2(u)$, $t = \frac{1}{2}$, for which $C_i(u)$, $i = 1, 2$ are not self intersecting, or even do not intersect each other in (b), and satisfy a simple metamorphosis relation, yet $C(u)$ self intersects.
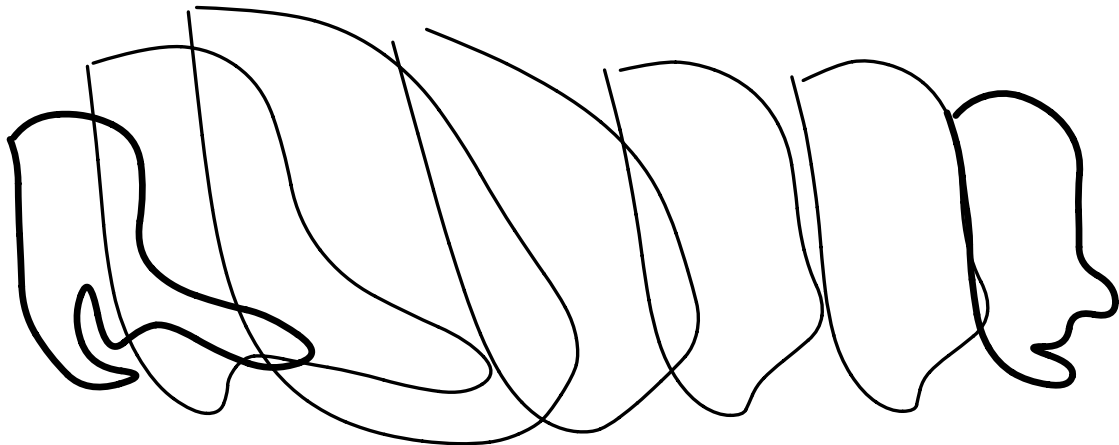

Figure 8: Two examples of combining edge cutting with simple metamorphosis relation testing. Compare with Figures 3 and 6.

in which the relation can be established can be replaced by simple linear metamorphosis as suggested in Section 2. If $C_1(u)$ and $C_2(u)$ are sufficiently similar that they satisfy Equation (5) from the start, the entire metamorphosis becomes a simple convex combination, completely preventing from the need to go through an intermediate linear segment. Figure 8 shows a metamorphosis example that combines $\mathcal{T}(u)$ verification with an edge cutting scheme.

The resulting metamorphosis, in all the above examples does not preserve another property that is sometimes considered important for a pleasing metamorphosis. The length of the inter-
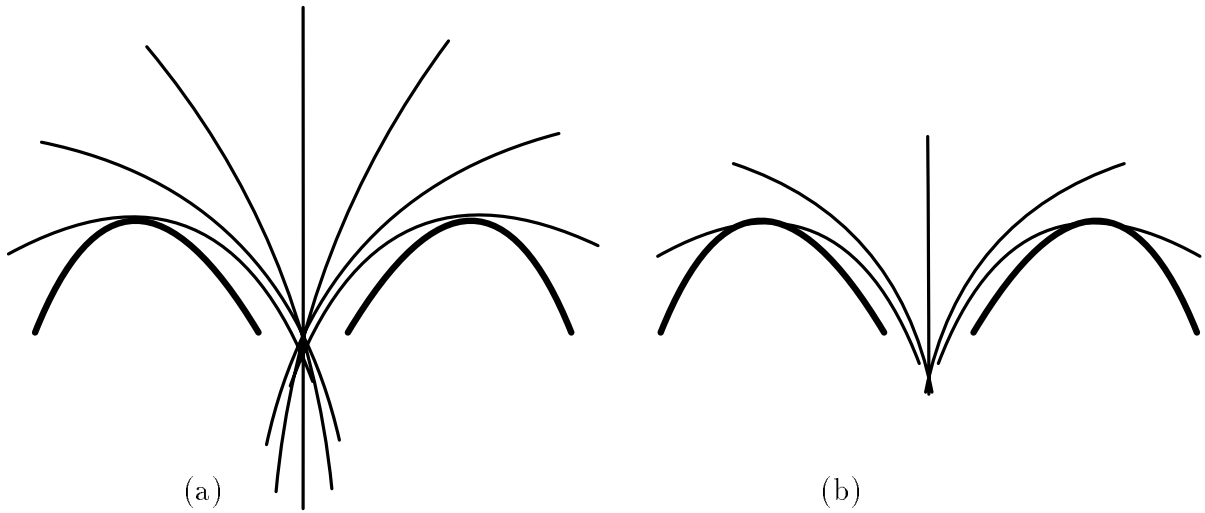
Figure 9: Two examples of metamorphosis of freeform curves using edge cutting. In (a) the lengths of the curves are preserved, while in (b) the bounding box dimensions is kept the same.

mediate curves is frequently required to be preserved while the general size of the curve should be preserved in other cases. One can scale the intermediate curves to preserve monotone change of the curves length, or alternatively one can attempt to preserve the general size by coercing a monotone change of the dimensions of the bounding box of the curves. Figure 9 shows two examples.

5 Extensions and Conclusions

The three presented methods of metamorphosis of curves can be extended to freeform surfaces, some with more effort than other. Clearly, convex combination of freeform surfaces (i.e. extending Equation 1 to surfaces), once in the same function space, is simple to implement. Surprisingly, results are frequently more appealing than convex combination based metamorphosis of curves. Due to the newly introduced dimension, the eyes and the brain are probably overwhelmed by the complexity and are is less sensitive to self intersections as in the curve case. Figure 10 shows several snapshots of an animation movie, that metamorphs a disc into a bottle, into a wine glass, and into a horn, all using the convex combination approach.

One might consider extending the edge cutting scheme to freeform surfaces, by considering each control point of the mesh, P_{ij}, as a apex of pyramid whose base is in the plane fitted

through $P_{i\pm1,j\pm1}$. The simple metamorphosis relation test, suggested in Equation 5, can be extended to surfaces, possibly correlating their normals,

$$\mathcal{N}(u,v) = \langle n_1(u,v), n_2(u,v) \rangle , \tag{6}$$

where $n_i(u,v)$ is the normal of surface $S_i(u,v)$.

We have examined three different methods to compute metamorphosis of freeform curves. The direct convex combination of two curves sharing the same function space, is the most simple one. Yet it is prone to self intersection and degeneracies. The multiresolution decomposition approach was also tested and was not found very successful. On the other hand, the edge cutting method combined with the simple metamorphosis relation test was found to be successful in creating nicer and more pleasing metamorphosis transitions of freeform curves.

It is clear that the self intersection free metamorphosis of freeform curves is a difficult problem that needs further investigation. Establishing correspondence between the two curves needs further study. Contemporary algorithm for the metamorphosis of piecewise linear polylines do require some manual interaction to define this correspondence. For freeform curves, the parametrization adds some difficulties into the metamorphosis process. The correspondence procedure should take into account the, usually different, parametrization of the two curves.

In a similar fashion, metamorphosis of freeform surfaces can be computed by a convex combination of corresponding control points of two surfaces sharing the same function space. Expanding the edge cutting scheme to general surfaces is feasible and should be explored as well as deriving metamorphosis between surfaces as a metamorphosis of the curves used to construct the two surfaces.

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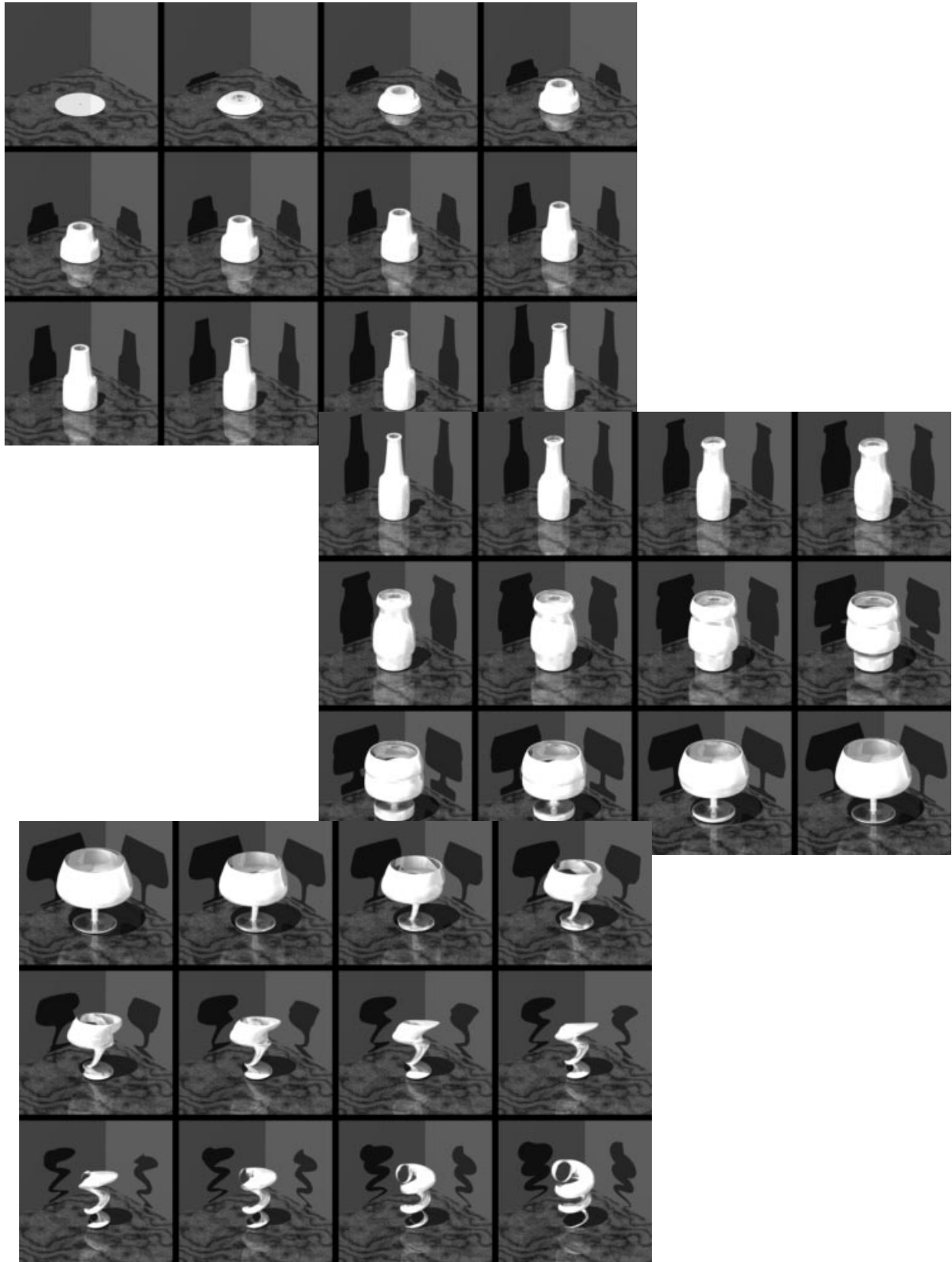
Figure 10: Snapshots from an animation movie of freeform surface metamorphosis using convex combination.

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