Representation of Graded Materials and Structures to Support Tolerance Specification for Additive Manufacturing Application

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Additive manufacturing (AM) has enabled control over heterogeneous materials and structures in ways that were not previously possible, including functionally graded materials and structures. This paper presents a novel method for representing and communicating heterogeneous materials and structures that include tolerancing of geometry and material together. The aim of this paper is to propose a means to specify nominal materials, nominal structures and allowable material variations in parts, including (a) explicit material and structural transitions (implying abrupt changes) and (b) functional transitions to support single and multiple material and structural behaviors (implying designed function-based gradients). The transition region combines bounded regions (volumes and surfaces) and material distribution and structural variation equations. Tolerancing is defined at two levels, that of the geometry including bounded regions and that of the materials. Material tolerances are defined as allowable material variations from nominal material fractions within a unit volume at a given location computed using material distribution equations. The method is described thorough several case studies of abrupt transitions, lattice-based transitions, and multimaterial and structural transitions.
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1 Introduction

Additive manufacturing (AM) merges design with manufacturing and merges material behaviors with complex geometries, creating novel parts while adhering to unique functional requirements. To benefit from the technological advantage availed by AM, new designs, process planning methods, tools, and representation schemes are rapidly being investigated [1–6]. The focus of this research is the representation (and communication) of complex parts with heterogeneous material structures. Though the principles in this paper are discussed in the context of heterogeneous materials, they readily apply to homogenous materials that may vary grain structures or other characteristics (e.g., intended porosity).

To produce functionally useful complex parts with heterogeneous materials, specification, control, and verification of variations (geometry and materials) are needed. As a first step, Ameta et al. [7–9] discussed gaps and possible solutions toward specification of geometric variations in parts designed for AM. Moving forward, the specification and verification of nominal materials and structures and allowable material and structure variations in parts pose challenges widely different than those of traditional geometry specifications.

Traditionally, material specification in design has essentially relied on standardized materials [9]. For each individual part in a product, a homogeneous material is specified. It is communicated that the material used for the part will be in accordance with material standards that specify material properties [10,11]. For single parts that incorporate heterogeneous material behaviors, this material specification scheme is inadequate.

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Parts incorporating heterogeneous material behavior have been manufactured by various AM processes for quite some time. Kumar et al. [12] demonstrated multimaterial powder deposition for sintering-based layered manufacturing. Khalil et al. [13] utilized the extrusion-based multimaterial deposition system for constructing bioactive polymer scaffolds. Willis et al. [14] combined materials and electronics to create optical interactive devices. Liu and Jang [15] developed an AM machine to create multimaterial parts using electrostatic imaging and lamination. Chen et al. [16] combined materials to create elastic and optical parts to meet different specifications. Church et al. [17] proposed a method to create electronic circuits and devices using AM. Jost et al. [18] developed wearable electronic devices using AM technology.

To close the gap between the ability to manufacture AM parts with heterogeneous material and structure behavior and the ability to design and specify (with tolerance) such parts, this research aims to develop a novel transition region specification. Section 2 discusses heterogeneous material models developed in the past, AM related complex structures, and tolerancing issues. Section 3 presents the transition region model developed in this research. Section 4 presents several case studies with graded materials and tolerances followed by graded structures with tolerances in Sec. 5.

2 Literature Review

The literature is described in two sections—heterogeneous materials and heterogeneous structures for AM and tolerancing.

For homogeneous materials, the specification is based on nominal material selection in the design stage. The conformation to the homogeneous material is specified as per American Society of Testing and Materials composition tests. Wherein variation from nominal is predefined in order to classify one material or alloy. The designer has no control over the material variations as per his design intent. AM techniques provide designers with finer control over the nominal composition and variation for nominal composition. These material composition specifications then need to be communicated to all AM stakeholders. The current material composition specification techniques cannot provide such a detailed specification of multimaterial nominal compositions and variations. The purpose of this research and related standards development is to meet these needs. Therefore, this literature review is focused more on heterogeneous materials and variation specification related to AM.

2.1 Heterogeneous Materials.

Modeling heterogeneous materials into geometry is a challenging problem that has crossed multiple domains. Here, the models developed for heterogeneous materials are discussed based on their intended use. The first type of model is developed for design- or analysis-based applications, e.g., computer-aided design (CAD) or finite element analysis. The second type of model is developed for use with AM.

2.1.1 Computer-Aided Design/Finite Element Analysis-Based Modeling. These types of models of multimaterial geometries were developed for (*a*) modeling and visualization and (*b*) conducting finite element analyses. A thorough review of these models is presented in Refs. [19] and [20]. Briefly, these types of models can be divided up into several categories, including voxels [21–23], volume meshes [24,25], functions (explicit [26–29] and implicit [30,31]), control features (volume, surface, edges [19,32–39], and points [40–43]), assemblies [25,44–46], cellular [47,48], and hybrids [49,50].

Voxel-type models represent the desired geometry with unit cubes aligned to a coordinate system. Heterogeneous materials are supported by assigning each cube with an appropriate material. These types of models are usually suited for visualization purposes.

Volume mesh-based models rely on a concept similar to voxels, but these are actual volumetric divisions of the part geometry. The volume meshes have a specified unit geometry but can be distorted to fit the part geometry. Furthermore, each unit mesh may incorporate multiple materials based on blending functions. The volume meshes can be utilized for finite element analysis.

Explicit function-based models use functions to model distributions of the heterogeneous material in the part geometry. These are very powerful methods that can be used for visualization and finite element analysis. The main drawback of explicit function-based methods is that the designer needs to use ad hoc methods to derive these functions to meet his needs, limiting the impact of specification and tolerance.

Control feature-based methods utilize functional needs of differing materials to assign control geometries for specific materials in the part. The overall distribution of materials in the part geometry is calculated based on predefined CAD procedures or a function.

Assembly-based models utilize combinations of parts of different materials to construct multimaterial parts. The combinations result in overlapping regions of materials that are then evaluated.

Cellular models are similar to assembly models but they utilize nonmanifold geometry (in lieu of parts with different materials) to construct multimaterial geometries. The nonmanifold geometry is combined with smoothening functions to create a geometry with smooth transitions between regions of different materials. Hybrid methods utilize any combination of the above methods to model heterogeneous material-based parts.

2.1.2 Additive Manufacturing-Based Representation of Geometry and Materials. AM processes require tessellated representations as source files for creating scan strategies. For single material models that use mesoscale responses to manufacture macroscale behaviors, simple geometry identification is sufficient. When addressing heterogeneous material-based parts, the source files must be able to handle multiple materials and the scan strategies must be able to interpret them. Tessellated representations currently leveraged by AM include STereoLithography (STL), Additive Manufacturing Format (AMF), three-dimensional (3D)-printing Manufacturing Format (3MF), etc. These will be discussed in the following paragraphs based on their representation capabilities for heterogeneous materials.

STL: The STereoLithographic format [51] is a file format that represents geometry as triangles. In a traditional sense, standalone STL does not support heterogeneous materials. Lei et al. [51] presented a nested STL shell-based method to represent density distributions of human bones.

AMF: Additive Manufacturing Format [52] is a file format that represents geometry as curved and straight triangles. AMF was created to overcome several drawbacks presented by the STL format with increased language expressivity. One of the advantages of AMF is to represent multimaterials. Regions can be defined in an AMF format using triangles, functions, or voxels [5]. These regions can then be associated with different materials or combination of materials as functions.

3MF: 3D manufacturing format is a file format that represents geometry as triangles. 3MF includes specifications of materials on objects and triangles [53]. Specification of heterogeneous materials on multiple objects or triangles of same object is feasible. However, to the best of the authors knowledge, functional distribution of materials is not feasible in 3MF.

Typical (CAD) systems are not built for modeling and representing heterogeneous materials-based geometry, though most do support representation of such geometries. The product data exchange standard [54] (standard for exchange of product data—STEP) has capabilities for indicating heterogeneous materials and gradients [19].

Table 1 compares the models and representation schemes discussed in Sec. 2 based on their material, CAD and AM requirements, and capabilities. As is evident, different model types were developed for heterogeneous materials modeling for procedural purposes, representational purposes, analysis purposes, and manufacturing purposes. None of the developed methods catered to the unique requirements of allocating variations in materials as discussed in this paper.

2.2 Heterogeneous Structures in Additive Manufacturing.

Many heterogeneous structures are being produced by AM processes. Lattices, topology optimized shapes, and organic structures are the most prominent heterogeneous structures. Lattice structures can also be combined with topology optimized shapes to generate conformal lattices with varying thicknesses [58–60]. These heterogeneous structures can be represented using voxel-based methods, functional equations, or other computational geometric techniques.

2.3 Tolerancing.

Proper functioning of a product relies on manufacturing the product within specifications, including allowable (usually geometric) variations. These allowable variations are called tolerances. Tolerance specification is the specification of the type and value of tolerances based on the standards (ASME Y14.5 [61] or ISO 1101 [62]). These standards provide a language

Table 1 Comparison of existing models for heterogeneous materials and geometry. Rating of 5 is most useful, and rating of 1 is least. Material capability A—single, B—multiple, and C—multiple with gradient functions

Literature	type	Mat.	Proc	Rep	FE	AM
	Voxel	B	No	Yes	3	3
[24,25]	Volume mesh	B	No	Yes	5	3
	Explicit functions	C			2	2
[33]	Control feature	C	Yes	Yes	3	2
[40–43]	Control point	C	Yes	Yes	3	2
[30,31,56]	Implicit functions	C	No	No	2	2
[35,44–46]	Assembly	C	No	No	2	2
[47,48]	Cellular	C	No	No	3	3
[49,50,55]	Hybrid	C	No	No	2	2
[51]	Tessellated	A	No	Yes	1	5
[57]	Curved tessellated	C	No	Yes	1	5
[52]	Tessellated	B	No	Yes	1	5

Note: FE stands for Finite element analysis.

to communicate acceptable 3D variations of geometric elements in a part from design to manufacturing and inspection. This language is called geometric dimensioning and tolerancing (GD&T). GD&T is based on mathematical representations of the variation of geometric elements and manufacturing knowledge bases [63,64]. GD&T is also a way of specifying design intent to prevent misrepresentation during production processes.

Despite advances in tolerancing geometry and dimensions, specifying allowable variations in materials for parts with heterogeneous materials has not been pursued. AM processes have the capability to produce intricate and complex shapes having functional gradients that are not feasible with traditional manufacturing processes. The aim of this paper is to propose a means to specify nominal property (materials and geometry) and allowable property (materials and geometry) variations in parts, including (*a*) explicit property transitions and (*b*) functional transitions to support single and multiple property (material and geometry) behaviors. A novel method called transition regions is proposed and discussed in Secs. 3 and 4.

3 Transition Regions—Definition

A *transition region* is defined as a bounded region (volume or surface) in the part where any locally controllable property or characteristic (material, geometry, etc.) is transitioning from one or more types to others. A *bounded volume* is defined as a volume subset within a part that is bounded by a set of connected or intersecting surfaces. A bounded volume is shown in Fig. 1 with local note "VR." In the part shown, there are two volumetric bounded regions. The first one shown is labeled as VR1 while the second one is shown transparent for clarity.

Each transition region is associated with a transition function. The transition function defines the nominal transition of the material or any other property. Each transition in the transition region is indicated with symbol m_i. The transition function for property m_i is defined by a function f_i. At any given location (x, y, z; Cartesian or otherwise) of the transition region, the required property m_i is computed using Eq. (1). Thus, the property m_i can be transformed to parametric space creating a *basis function* for the property (Eq. (2*a*)). In any given location, the sum of all properties m_i should satisfy Eq. (2*b*), i.e., they form a *partition of unity*. Furthermore, tolerance on the nominal value of the property m_i is t_{mi} and will be computed as m_i X t_{mi}. Besides the tolerance on the property transitions, there will be geometric tolerances on the transition regions as well

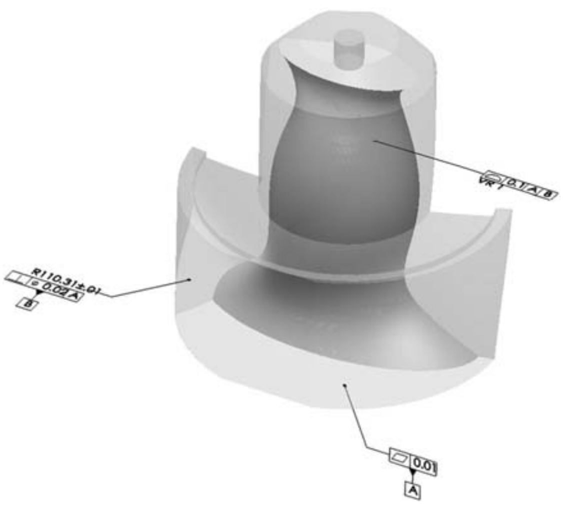

Fig. 1 Bounded volume shown with local note VR and profile tolerance

$$m_i = \frac{f_i(x,y,z)}{\sum f_i(x,y,z)} \tag{1}$$

$$m_i(u,v,w\ldots) = \Theta m_i(x,y,z\ldots) \tag{2a}$$

$$\sum_i^n m_i = 1 \tag{2b}$$

In trivial cases, when one of the material compositions is noncritical, for n–1 materials $m_i = f_i$ and for nth material

$$m_n = 1 - \sum_1^{n-1} f_i$$

In nontrivial cases, the material compositions are normalized by adjusting each material composition based on the function value at a given location. Part/assembly model definitions will include the functions and material specification codes. STEP AP 242 [65], for example, can include such equations. Efforts are ongoing to incorporate presentation scheme described in this paper and related representation scheme to be included in STEP AP 242 and ASME Y14.41 [66].

4 Graded Materials

Gradient structure can be specified using transition regions. Application of transition regions will be demonstrated using several cases studies in this section. Although fractions and/or percentage is used to specify nominal properties and associated tolerances, the method presented is applicable even if absolute values are used. Furthermore, to keep the discussion simple, any effect of geometric tolerances is abstained until Sec. 6.

4.1 Case 1: Two Material Transition in Simple Part.

In case 1, three different types of examples are considered. In the first example, a material transition is defined using a transit ion region. In the second example, a transition region is not defined, instead two volume regions are indicated with different materials and geometric tolerances. In the third example, a material transition is defined using a transition region with geometric tolerances.

4.1.1 Material Transition Region Only With the Linear Function on One Parameter. An example of a material gradient is shown in Fig. 2. The part has three volumetric bounded regions. The first region has material MAT1, and the third region has material MAT2. The second region is defined as transition region from MAT1 to MAT2. In the transition region, the transition function for MAT1 and MAT2 is specified as $f_2 = z - 10$ and $f_1 = 15 - (z - 10)$. The tolerance on MAT2 and MAT1 is given in

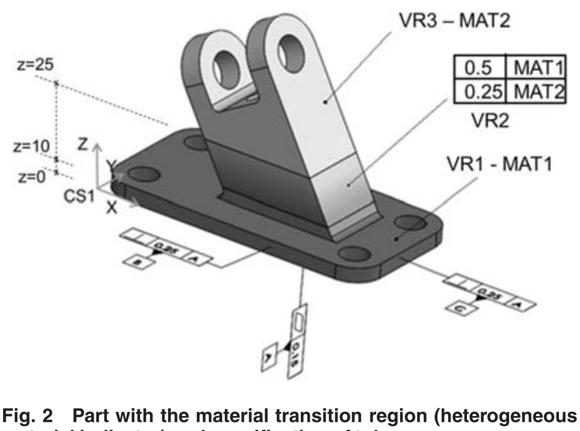

Fig. 2 Part with the material transition region (heterogeneous material indicator) and specification of tolerance

Table 2 Example of material gradient values (shown as % instead of fractions)

Z mm	Nominal MAT1	Nominal MAT2	Tolerance on fraction of MAT1	Tolerance on fraction of MAT2
10	0%	100%	+25%	−12.5%
15	33%	66%	±25%	±12.5%
20	66%	33%	±25%	±12.5%
25	100%	0%	−25%	+12.5%

Fig. 2 as 0.25 and 0.5 in a stacked control frame. Following Eqs. (1) and (2) from Sec. 3, the nominal material composition and its respective tolerances along the z-axis (vertical direction in Fig. 2) can be computed (Table 2). The basis functions m_i can be derived from f_i as

$$m_2 = \frac{z-10}{15} = t$$
$$m_1 = 1 - \frac{z-10}{15} = 1 - t$$

Note that the actual value of the material composition in a minimal measurable volume in the part space cannot exceed 1 but can be less than 1(owing to voids).

Figure 3 describes the nominal values of materials along the z axis with their tolerance zones and a set of allowable values along the z axis for MAT1 and MAT2. The sum of values for MAT1 and MAT2 at a given z coordinate cannot exceed one (following equation [2]). For a chosen allowable value of MAT1 (black triangle at a given z coordinate), a corresponding maximum allowable value of MAT2 (gray circle at the same z coordinate) is shown in the graph. Any value of MAT2 below what is shown in the graph within the tolerance zone is also acceptable.

Figures 4 and 5 demonstrate cases when MAT1 is measured at four places, and then, acceptable zone of values for MAT2 is shown and vice versa. In Fig. 4, MAT2 is assumed to be measured first at z values of 10, 15, 20, and 25. The measured values of MAT2 are marked as gray circles. The resultant acceptable zone for MAT1 is shown shaded as gray. In Fig. 5, MAT1 is assumed to be evaluated at z values first (at z values of 10, 15, 20, and 25). The measured values of MAT1 are marked as black triangles. The resultant acceptable zone for MAT2 is shown shaded as gray. As is evident, due to differences in material functions and the tolerance on material variations, different amount of allowable variations for each material is encountered along the z-axis for VR2.

4.1.2 Material Transition Region Only With the Nonlinear Function on Two Parameters. Instead of the linear functions for MAT1 and MAT2 distribution in VR2 of Fig. 2, let us assume that the functions are

$$f_1 = xz + 10\sin((z-10)10) + 10\cos(10x) \tag{3}$$

$$f_2 = 1250 - xz - 10\sin((z-10)10) - 10\cos(10x) \tag{4}$$

Then, the nominal material fractions in VR2 for MAT1 and MAT2 can be computed as shown in Figs. 6(*a*) and 6(*b*), respectively. The tolerance zones would be offset surfaces in both directions, truncated at material fractions 1 and 0.

4.2 Case 2: Multimaterial Transition. The third example demonstrates multimaterial transition in a single volume region. Figure 7 shows a part with four volume regions labeled VR1, VR2, VR3, and VR4. VR1, VR2, and VR3 have material specifications MAT1, MAT2, and MAT3. In Fig. 7, MAT1, MAT2, and MAT3 are shown in medium, light, and dark gray, respectively. VR1 is shown transparent to show the geometry of VR4. VR4 is the volume region where three materials transition based on the functions described in Eqs. (5)–(7). VR4 is a cylinder with semispherical end within VR3. The cylindrical coordinate system is utilized with origin at the center of the circular face between VR2 and VR4. The X-axis is along the axis of VR4, and r is along the radius of VR4 ($r_{max} = 15$). The length of VR4 is 60 ($x = 0$ to $x = 60$). The cylindrical part is from $x = 0$ to $x = 45$ while the rest is semisphere with a radius of 15.

These functions are numerically computed and then normalized based on Eq. (2). The normalized material distributions of MAT1, MAT2, and MAT3 for VR4 are shown in Fig. 8(*a*). MAT1 is from VR1 and therefore has higher concentrations at r close to 15. MAT2 is from VR2 and therefore has higher concentrations at r close to 0. MAT 3 is from VR3 and therefore has higher concentrations in a circular region $x > 45$ and $r > 12$. Figure 8(*b*) shows a cross section of the complete part with VR1 (MAT1—medium), VR2 (MAT2—light), VR3 (MAT 3—dark), and the material transitions in VR4. Each material shade (medium, light, and dark) is

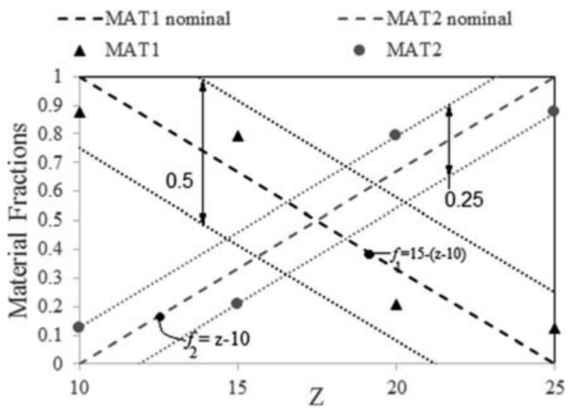

Fig. 3 Nominal, limit, and acceptable material fractions along the *z*-axis for VR2, based on the equations embedded in the part model (The unit of *z* is inches)

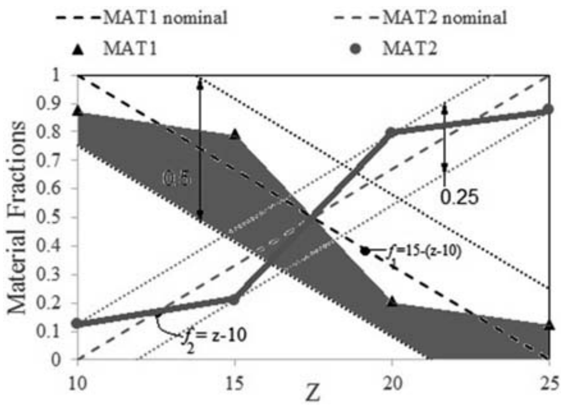

Fig. 4 Implication of material tolerances. Given material fractions of MAT2, the acceptable material fractions of MAT1 marked as a gray area. Any value of MAT1 below the upper line marked with black triangles will lead to acceptable void fractions (The unit of z is inches).

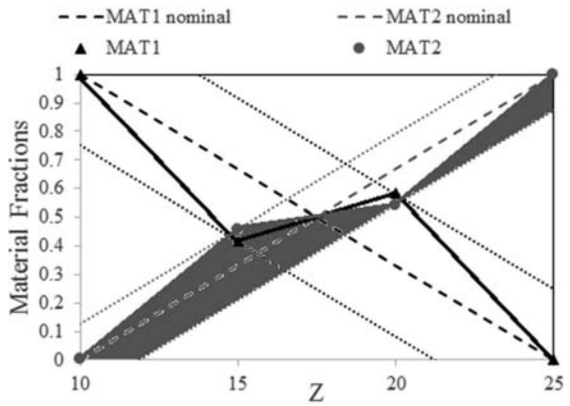

Fig. 5 Implication of material tolerances. Given material fractions of MAT1, the acceptable material fractions of MAT2 marked as a gray area. Any value of MAT2 below the upper line marked with gray circles will lead to acceptable void fractions (The unit of z is inches).

mixed to form the final shade in VR4 based on the normalized material distributions computed from Eqs. (5)–(7)

$$f(\text{Mat1}) = \begin{cases} \dfrac{r}{x-45} & x \le 45 \\ \dfrac{x-45}{r} & 60 > x > 45 \end{cases} \tag{5}$$

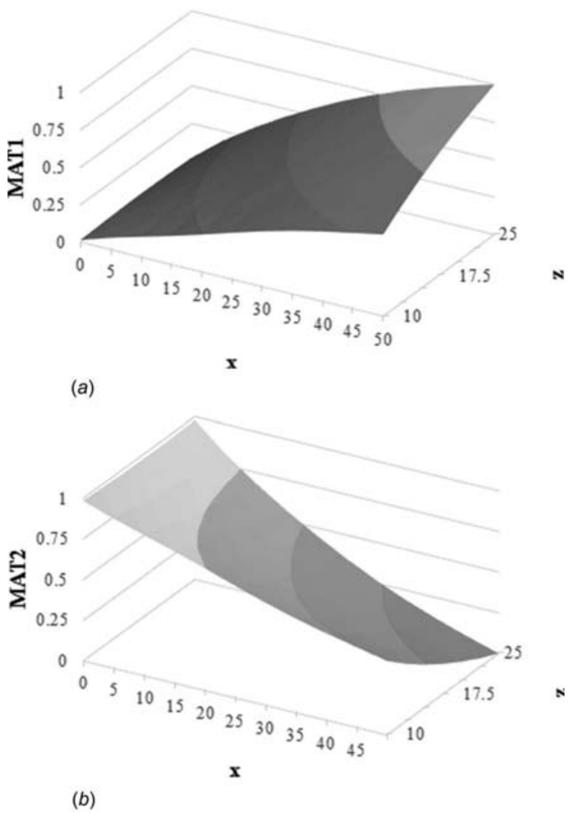

Fig. 6 Material fractions for MAT1 and MAT2 (The unit of x and z is inches): (*a*) MAT1 fractions along *x* and *z* axes and (*b*) MAT2 fractions along *x* and *z* axes

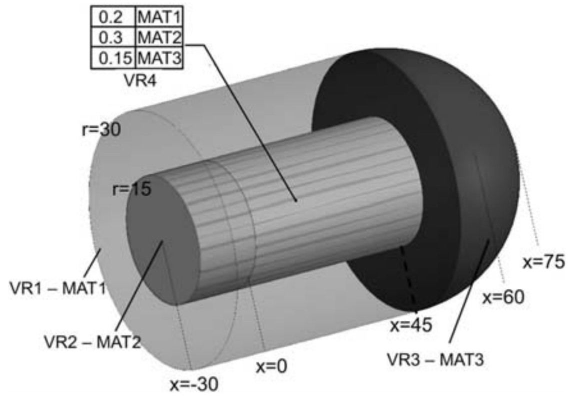

Fig. 7 A part with four volume regions with three material specifications

$$f(\text{Mat2}) = \begin{cases} \dfrac{60}{x} & x \ge 30 \\ \dfrac{60-x}{1+r} & x < 30 \end{cases} \tag{6}$$

$$f(\text{Mat3}) = \begin{cases} \sqrt{(x-45)^2 + r^2} & \text{if} \begin{cases} r \ge 4;\, 60 \ge x \ge 45 \\ \sqrt{(x-45)^2 + r^2} \le 15 \end{cases} \\ \dfrac{\sqrt{(x-45)^2 + r^2}}{2} & \text{if} \begin{cases} r < 4;\, 60 \ge x \ge 45 \\ \sqrt{(x-45)^2 + r^2} \le 15 \end{cases} \end{cases} \tag{7}$$

Figure 9 shows three possible extreme changes in material compositions for VR4 based on the allowable variations indicated in Fig. 7. Figure 9(*a*) shows composition of MAT 1, MAT2, and MAT3 changed by −0.1, +0.15, and −0.075, respectively, according to the material tolerances. Figure 9(*b*) shows composition of MAT 1, MAT2, and MAT3 changed by +0.1, −0.15, and +0.075, respectively. The unit area where the material composition of any material is different from nominal (Fig. 8) by fraction 0.04 is highlighted with black lines. Figure 9(*c*) shows the composition of MAT 1, MAT2, and MAT3 changed by +0.1, 0.0, and +0.075, respectively. The position of black grids indicates the impact of material tolerances on material compositions in VR4. A designer, who designs heterogeneous material-based part, must evaluate the impact of material tolerances on the function of the part or volume region as needed.

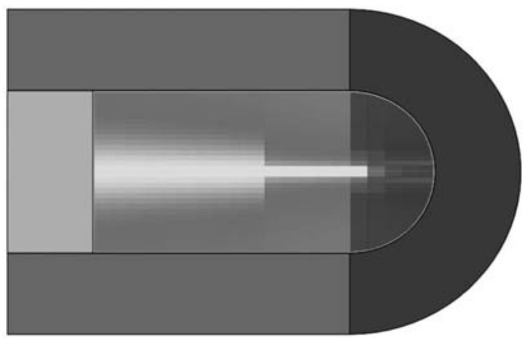

Fig. 8 Nominal material distributions as computed from Eqs. (5)–(7). Cross section of the part from Fig. 7 showing the material composition encoded as shades.

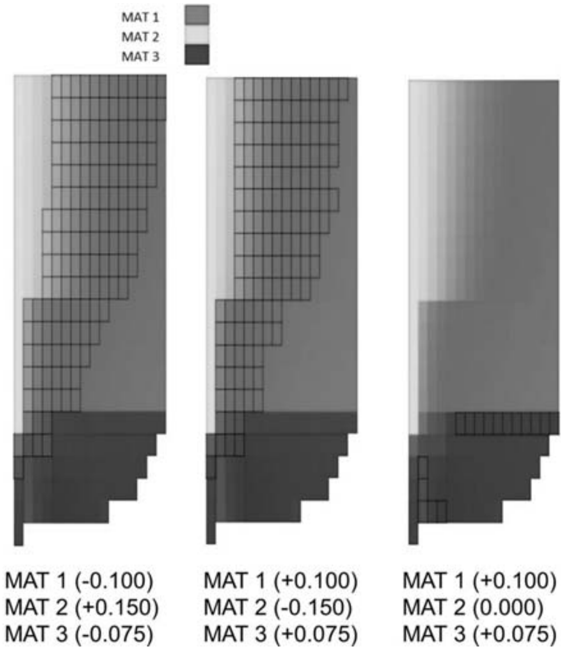

Fig. 9 Impact of material distribution based on the allowable material variations from Fig. 7

5 Geometric and Graded Structures

Complex structures can be created using AM technologies. These structures can be defined using geometry or functions for materials and voids. Function-based representation will be discussed as graded structures while geometry-based representation will be discussed as geometric structures.

5.1 Geometric Structures. Complex geometry can be generated in CAD systems to suit a variety of functions. Based on the discussion in Sec. 4, lattice-based geometric structures within the bounded volume are presented in this section. In such geometric structures, variations in geometry are represented by the geometry itself. These variations could be abrupt or could follow a predefined grading scheme. Material variations in these geometric structures need to be represented using transition regions.

Figure 10 identifies a part with five lattice-based volume regions. Volume region 2 is the transition region for material MAT1 in VR1 and material MAT2 in VR3. Volume region 4 is the transition region for material MAT2 in VR3 to material MAT3 in VR5.

The material transition functions can be applied as needed based on the discussion in Sec. 4.1.

5.2 Graded Structures. Complex geometry that is varied using mathematical functions is discussed in this section. Two types of such geometric elements will be presented—lattice and porosity.

A two-dimensional example of lattice is presented in Fig. 11. Figure 11(*a*) represents a simple rectangular lattice enclosed within a part boundary. This uniform lattice is generated using Eqs. (8)–(10). In these equations, x and y represent coordinates, s_1 represents the slope of the lines, t represents constant thickness while parameter c_1 is used to create a pattern of the line to generate the lattice. Figure 11(*b*) incorporates varying thicknesses of the lattice element. This is accomplished by modifying the thickness parameter t using Eq. (11). Figure 11(*c*) represents conformal

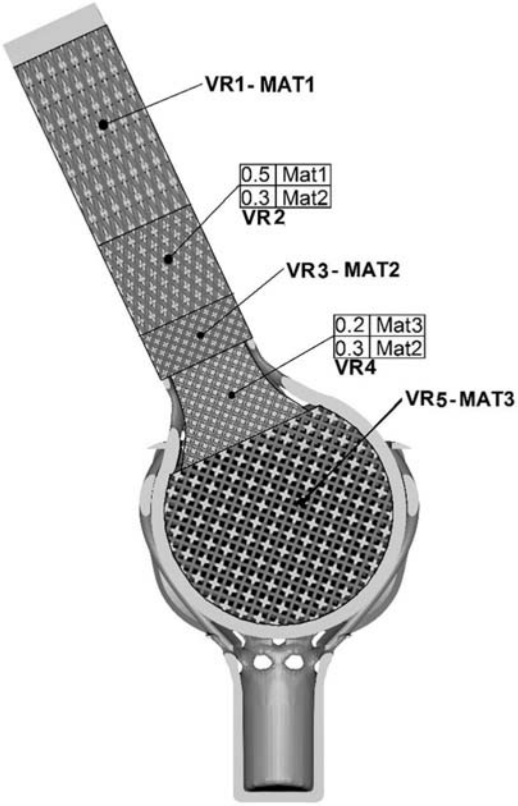

Fig. 10 Example of material transition specification with tolerance between bounded lattice regions

lattice. This is accomplished by varying the slope parameters s_1 with respect to x and y as shown in Eq. (12).

$$\left.\begin{aligned} y+\frac{t}{2} &\leq s_1 x + c_1 \\ y-\frac{t}{2} &\geq s_1 x + c_1 \end{aligned}\right\} \tag{8}$$

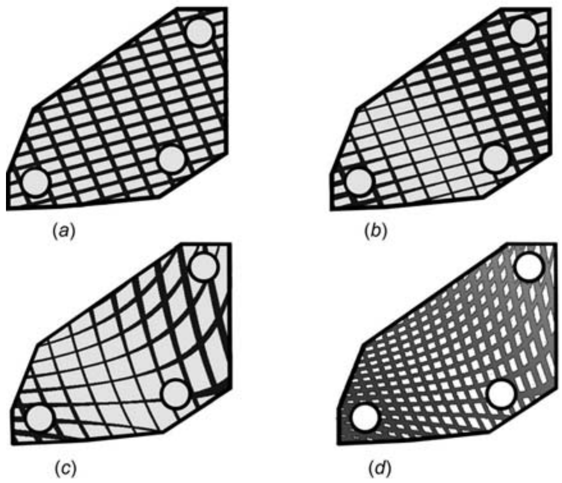

Fig. 11 Four cases of functionally defined graded structures: (*a*) simple lattice, (*b*) thickness graded lattice, (*c*) conformal lattice with the graded thickness, and (*d*) conformal lattice with the graded thickness and material

Table 3 Porosity design functions

Size	Position
Uniform (1,10)	x = Uniform (0, xMax); y = Uniform (0, yMax)
Uniform (1,10)	x = Gaussian(μ, σ); y = Gaussian(μ, σ)
Uniform (1,10)	Y = ±2*cos(x) + noise

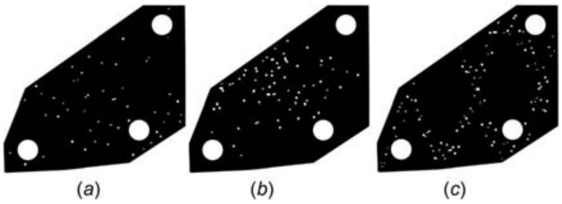

Fig. 12 Three cases of desired porosity in a part: (*a*) uniformly distributed, (*b*) normally distributed, and (*c*) following a function

$$\left.\begin{aligned} y+\frac{t}{2} &< -\frac{x}{s_1}+c_1 \\ y-\frac{t}{2} &> -\frac{x}{s_1}+c_1 \end{aligned}\right\} \tag{9}$$

$$c_1 = 4 + \mathrm{mod}\left(\frac{w}{3t}\right) \tag{10}$$

$$t = 1 + \frac{\cos\sqrt{x^2+y^2}}{4} \tag{11}$$

$$s_1 = \frac{1}{1+x} \tag{12}$$

Similar to lattice structures based on functions, the porous structure can also be generated. Figure 12 and Table 3 demonstrate three cases of varying porosities. Figure 12(*a*) shows random distribution of voids with varying sizes. Figure 12(*b*) shows Gaussian distribution of voids with varying size. Figure 12(*c*) shows voids following a defined function within the part.

5.3 Graded Structures With Graded Materials. Graded structures can be combined with graded materials. As an example, two material gradients are present in the lattice of Fig. 11(*d*) using Eqs. (13) and (14) for the material in light and dark, respectively. The tolerance for material fractions in the transition regions for light is 0.2 and dark is 0.15 (see Fig. 13)

$$f_1 = 0.9\sqrt{x^2+y^2} \tag{13}$$

$$f_2 = 1 - 0.9\sqrt{x^2+y^2} \tag{14}$$

6 Combined Geometric and Material Tolerances

When no material transition is specified between two heterogeneous materials in a part, the geometric tolerances on the volumetric regions will serve as the transition region. In such a scenario, the material may vary abruptly or transit or a combination of both, in the tolerance zone, in order to transition from one material volume to another (Fig. 14(*a*)).

When both the transition region and geometric tolerance are specified, the nominal material functions and related material tolerances are applicable in the transition region (Fig. 14(*b*)). The geometric tolerance zones can have material variation as needed to finalize the transition.

7 Conclusion and Discussion

AM has enabled exploration of design spaces with complex geometry and heterogeneous materials. Design of heterogeneous material-based complex geometries entails specification and tolerancing of materials and geometry to specify allowable variations. This paper demonstrated a method using transition regions to tolerance material compositions and related geometry in a part. Case studies elucidated the applicability of the proposed transition region method. Although, the application is mainly demonstrated on heterogeneous materials, the transitions can apply to homogeneous materials whose mesoscale characteristics can be designed to achieve effects similar to heterogeneous materials. This method is currently being explored for standardization under ASME Y14.46 subcommittee. Future work includes (*a*) application to the functionally graded material and product design, (*b*) integration with the finite element analysis, and (*c*) exploring lattice continuity in multilattice multimaterial-based designs.

The focus of this paper has been to describe tools that AM designer can use to present material fraction tolerances for parts with functionally graded materials. The issue of how a designer arrives at acceptable nominal material gradients and acceptable material fraction variations is a much bigger challenge involving

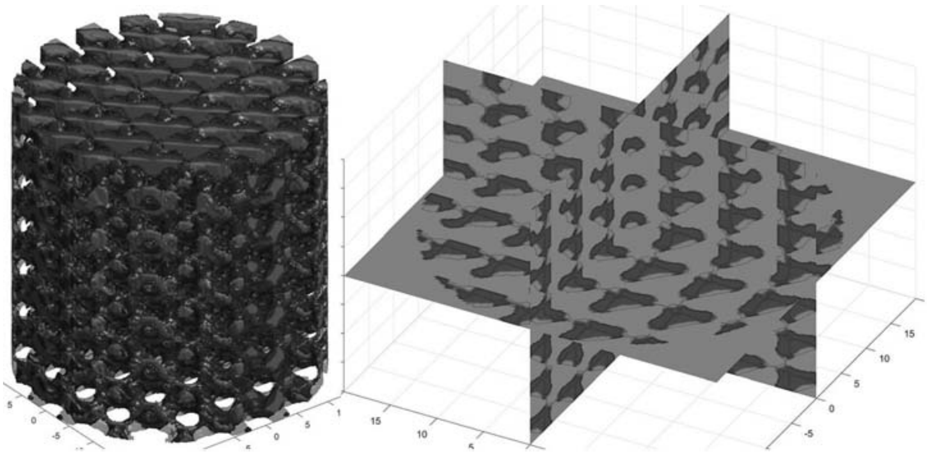

Fig. 13 Cylindrical lattice composed of two materials as a function of *x*, *y*, and *z*

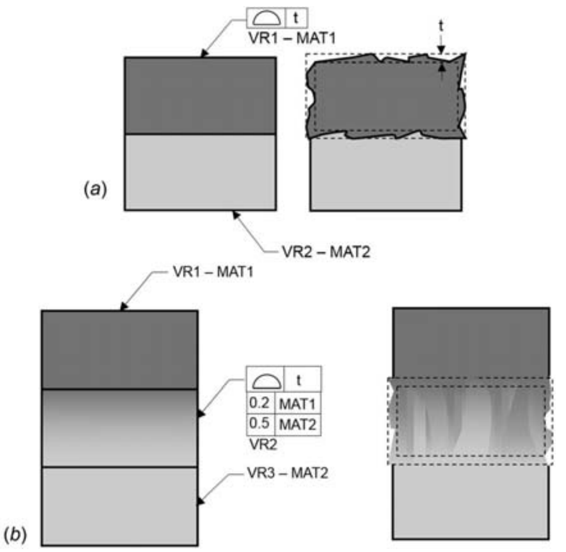

Fig. 14 (*a*) Geometrical profile tolerance on VR1 with its tolerance zone and (*b*) geometrical profile tolerance with nominal material and material tolerance for VR2. Shade and boundary variations indicate interactions of two tolerance zones.

iterative design, AM heterogeneous material capability studies, heterogeneous material analysis/simulation, and function-based design studies. AM heterogeneous material capability studies may include studying the material variations across layers, the effect of build directions, layer thickness, etc.

References

[1] Rosen, D. W., 2007, "Computer-Aided Design for Additive Manufacturing of Cellular Structures," Comput.-Aided Des. Appl., **4**(5), pp. 585–594.

[2] Gibson, I., Rosen, D. W., and Stucker, B., 2010, *Additive Manufacturing Technologies*, Springer, New York.

[3] Wong, K. V., and Hernandez, A., 2012, "A Review of Additive Manufacturing," ISRN Mech. Eng., **2012**, p. 208760.

[4] Brackett, D., Ashcroft, I., and Hague, R., 2011, "Topology Optimization for Additive Manufacturing," Solid Freeform Fabrication Symposium, Austin, TX, Aug. 8–10, pp. 348–362.

[5] Hiller, J. D., and Lipson, H., 2009, "STL 2.0: A Proposal for a Universal Multi-Material Additive Manufacturing File Format," Solid Freeform Fabrication Symposium, Austin, TX, Aug. 3–5, pp. 266–278.

[6] Gibson, I., Rosen, D., and Stucker, B., 2014, *Additive Manufacturing Technologies: 3D Printing, Rapid Prototyping, and Direct Digital Manufacturing*, Springer, New York.

[7] Ameta, G., Lipman, R., Moylan, S., and Witherell, P., 2015, "Investigating the Role of Geometric Dimensioning and Tolerancing in Additive Manufacturing," ASME J. Mech. Des., **137**(11), p. 111401.

[8] Witherell, P., Herron, J., and Ameta, G., 2016, "Towards Annotations and Product Definitions for Additive Manufacturing," Procedia CIRP, **43**, pp. 339–344.

[9] Ameta, G., Moylan, S. P., and Witherell, P. W., 2015, "Challenges in Tolerance Transfer for Additive Manufacturing," Summer Topical Meeting of American Society of Precision Engineering, Raleigh, NC, July 8–10.

[10] GRANTA, 2016, "GRANTA MI:Materials Gateway for Creo," GRANTA, accessed Nov. 2, 2016, http://www.grantadesign.com/products/mi/proe/

[11] ASTM, 2015, "Standard Guide for Identification of Metals and Alloys in Computerized Material Property Databases," ASTM International, West Conshohocken, PA.

[12] Kumar, P., Santosa, J. K., Beck, E., and Das, S., 2004, "Direct-Write Deposition of Fine Powders Through Miniature Hopper-Nozzles for Multi-Material Solid Freeform Fabrication," Rapid Prototyp. J., **10**(1), pp. 14–23.

[13] Khalil, S., Nam, J., and Sun, W., 2005, "Multi-Nozzle Deposition for Construction of 3D Biopolymer Tissue Scaffolds," Rapid Prototyp. J., **11**(1), pp. 9–17.

[14] Willis, K., Brockmeyer, E., Hudson, S., and Poupyrev, I., 2012, "Printed Optics: 3D Printing of Embedded Optical Elements for Interactive Devices," 25th Annual ACM Symposium on User Interface Software and Technology, Cambridge, MA, Oct. 7–12, pp. 589–598.

[15] Liu, J., and Jang, B. Z., 2004, "Layer Manufacturing of a Multi-Material or Multi-Color 3-D Object Using Electrostatic Imaging and Lamination," U.S. Patent No. US20020145213A1.

[16] Chen, D., Levin, D. I., Didyk, P., Sitthi-Amorn, P., and Matusik, W., 2013, "Spec2Fab: A Reducer-Tuner Model for Translating Specifications to 3D Prints," ACM Trans. Graph. TOG, **32**(4), p. 135.

[17] Church, K. H., Tsang, H., Rodriguez, R., Defembaugh, P., and Rumpf, R., 2013, "Printed Circuit Structures, the Evolution of Printed Circuit Boards," IPC APEX EXPO Conference, San Diego, CA, Feb., pp. 19–21.

[18] Jost, K., Stenger, D., Perez, C. R., McDonough, J. K., Lian, K., Gogotsi, Y., and Dion, G., 2013, "Knitted and Screen Printed Carbon-Fiber Supercapacitors for Applications in Wearable Electronics," Energy Environ. Sci., **6**(9), pp. 2698–2705.

[19] Kou, X. Y., and Tan, S. T., 2007, "Heterogeneous Object Modeling: A Review," Comput.-Aided Des., **39**(4), pp. 284–301.

[20] Patil, L., Dutta, D., Bhatt, A. D., Jurrens, K., Lyons, K., Pratt, M. J., and Sriram, R. D., 2000, "Representation of Heterogeneous Objects in ISO 10303 (STEP)," ASME International Mechanical Engineering Congress and Exposition, Orlando, FL, Nov. 5–8.

[21] Zhang, X.-J., Chen, K.-Z., and Feng, X.-A., 2004, "Optimization of Material Properties Needed for Material Design of Components Made of Multi-Heterogeneous Materials," Mater. Des., **25**(5), pp. 369–378.

[22] Cho, J. R., and Ha, D. Y., 2002, "Optimal Tailoring of 2D Volume-Fraction Distributions for Heat-Resisting Functionally Graded Materials Using FDM," Comput. Methods Appl. Mech. Eng., **191**(29–30), pp. 3195–3211.

[23] Hu, Y., Blouin, V. Y., and Fadel, G. M., 2008, "Design for Manufacturing of 3D Heterogeneous Objects With Processing Time Consideration," ASME J. Mech. Des., **130**(3), p. 031701.

[24] Jackson, T. R., 2000, "Analysis of Functionally Graded Material Object Representation Methods," Doctoral dissertation, Massachusetts Institute of Technology, Cambridge, MA.

[25] Liu, H., Cho, W., Jackson, T. R., Patrikalakis, N. M., and Sachs, E. M., 2000, "Algorithms for Design and Interrogation of Functionally Gradient Material Objects," ASME Paper No. DAC-14278.

[26] Zhu, F., 2004, "Visualized CAD Modeling and Layered Manufacturing Modeling for Components Made of a Multiphase Perfect Material," Master's thesis, University of Hongkong, Hongkong, China.

[27] Shin, K.-H., and Dutta, D., 2001, "Constructive Representation of Heterogeneous Objects," ASME J. Comput. Inf. Sci. Eng., **1**(3), pp. 205–217.

[28] Elishakoff, I., Gentilini, C., and Viola, E., 2005, "Three-Dimensional Analysis of an All-Round Clamped Plate Made of Functionally Graded Materials," Acta Mech., **180**(1–4), pp. 21–36.

[29] Eraslan, A. N., and Akis, T., 2006, "On the Plane Strain and Plane Stress Solutions of Functionally Graded Rotating Solid Shaft and Solid Disk Problems," Acta Mech., **181**(1–2), pp. 43–63.

[30] Pasko, A., Adzhiev, V., Schmitt, B., and Schlick, C., 2001, "Constructive Hypervolume Modeling," Graph. Models, **63**(6), pp. 413–442.

[31] Rvachev, V. L., Sheiko, T. I., Shapiro, V., and Tsukanov, I., 2001, "Transfinite Interpolation Over Implicitly Defined Sets," Comput. Aided Geom. Des., **18**(3), pp. 195–220.

[32] Siu, Y. K., and Tan, S. T., 2002, "'Source-Based' Heterogeneous Solid Modeling," Comput.-Aided Des., **34**(1), pp. 41–55.

[33] Biswas, A., Shapiro, V., and Tsukanov, I., 2004, "Heterogeneous Material Modeling With Distance Fields," Comput. Aided Geom. Des., **21**(3), pp. 215–242.

[34] Park, S.-M., Crawford, R. H., and Beaman, J. J., 2001, "Volumetric Multi-Texturing for Functionally Gradient Material Representation," Sixth ACM Symposium on Solid Modeling and Applications, Atlanta, Georgia, pp. 216–224.

[35] Liu, H., Maekawa, T., Patrikalakis, N. M., Sachs, E. M., and Cho, W., 2004, "Methods for Feature-Based Design of Heterogeneous Solids," Comput.-Aided Des., **36**(12), pp. 1141–1159.

[36] Samanta, K., and Koc, B., 2005, "Feature-Based Design and Material Blending for Free-Form Heterogeneous Object Modeling," Comput.-Aided Des., **37**(3), pp. 287–305.

[37] Bhashyam, S., Hoon Shin, K., and Dutta, D., 2000, "An Integrated CAD System for Design of Heterogeneous Objects," Rapid Prototyping J., **6**(2), pp. 119–135.

[38] Wei, H., Wang, Y., and Rosen, D. W., 2017, "A Multiscale Materials Modeling Method With Seamless Zooming Capability Based on Surfacelets," ASME J. Comput. Inf. Sci. Eng., **17**(2), p. 021007.

[39] Wei, H., Wang, Y., and Rosen, D. W., 2016, "Material Feature Representation and Identification With Composite Surfacelets," J. Comput. Des. Eng., **3**(4), pp. 370–384.

[40] Huang, J., Fadel, G. M., Blouin, V. Y., and Grujicic, M., 2002, "Bi-Objective Optimization Design of Functionally Gradient Materials," Mater. Des., **23**(7), pp. 657–666.

[41] Qian, X., and Dutta, D., 2003, "Design of Heterogeneous Turbine Blade," Comput.-Aided Des., **35**(3), pp. 319–329.

[42] Hua, J., He, Y., and Qin, H., 2004, "Multiresolution Heterogeneous Solid Modeling and Visualization Using Trivariate Simplex Splines," Ninth ACM Symposium on Solid Modeling and Applications, Genoa, Italy, June 9–11, pp. 47–58.

[43] Cohen, W. M. E., "Representation and Extraction of Volumetric Attributes Using Trivariate Splines: A Mathematical Framework," Sixth ACM Symposium on Solid Modeling and Applications.

[44] Kumar, V., and Dutta, D., 1998, "An Approach to Modeling & Representation of Heterogeneous Objects," ASME J. Mech. Des., **120**(4), pp. 659–667.

[45] Sun, W., and Hu, X., 2002, "Reasoning Boolean Operation Based Modeling for Heterogeneous Objects," Comput.-Aided Des., **34**(6), pp. 481–488.

[46] Qian, X., and Dutta, D., 2003, "Heterogeneous Object Modeling Through Direct Face Neighborhood Alteration," Comput. Graph., **27**(6), pp. 943–961.

[47] Cavalcanti, P. R., Carvalho, P. C. P., and Martha, L. F., 1997, "Non-Manifold Modelling: An Approach Based on Spatial Subdivision," Comput.-Aided Des., **29**(3), pp. 209–220.

[48] Cheng, J., and Lin, F., 2005, "Approach of Heterogeneous Bio-Modeling Based on Material Features," Comput.-Aided Des., **37**(11), pp. 1115–1126.

[49] Chen, M., and Tucker, J. V., 2000, "Constructive Volume Geometry," *Computer Graphics Forum*, Blackwell Publishers, Boston, MA, pp. 281–293.
[50] Adzhiev, V., Kartasheva, E., Kunii, T., Pasko, A., and Schmitt, B., 2002, "Hybrid Cellular-Functional Modeling of Heterogeneous Objects," J. Comput. Inf. Sci. Eng., **2**(4), pp. 312–322.
[51] 3D Systems Inc., 1989, "StereoLithography Interface Specification," 3D Systems publications, Valencia, CA.
[52] ISO, 2013, "Standard Specification for Additive Manufacturing File Format (AMF) Version 1.1," Geneva, Switzerland, Standard No. ISO/ASTM 52915.
[53] 3MF Consortium, 2015, "3D Manufacturing Format—Core Specification & Reference Guide," 3MF Consortium, accessed Nov. 1, 2016, http://3mf.io/wp-content/uploads/2015/04/3MFcoreSpec_1.0.1.pdf
[54] ISO, 2007, "Industrial Automation Systems and Integration—Product Data Representation and Exchange—Part 238: Application Protocol: Application Interpreted Model for Computerized Numerical Controllers," International Standards Organization, Geneva, Switzerland, Standard No. ISO 10303-238:2007.
[55] Kou, X. Y., and Tan, S. T., 2007, "A Systematic Approach for Integrated Computer-Aided Design and Finite Element Analysis of Functionally-Graded-Material Objects," Mater. Des., **28**(10), pp. 2549–2565.
[56] Pasko, A., Adzhiev, V., Sourin, A., and Savchenko, V., 1995, "Function Representation in Geometric Modeling: Concepts, Implementation and Applications," Vis. Comput., **11**(8), pp. 429–446.
[57] Lei, S., Frank, M. C., Anderson, D. D., and Brown, T. D., 2014, "A Method to Represent Heterogeneous Materials for Rapid Prototyping: The Matryoshka Approach," Rapid Prototyping. J., **20**(5), pp. 390–402.
[58] Savio, G., Meneghello, R., and Concheri, G., 2018, "Geometric Modeling of Lattice Structures for Additive Manufacturing," Rapid Prototyping J., **24**(4), pp. 351–360.
[59] Terriault, P., and Brailovski, V., 2018, "Modeling and Simulation of Large, Conformal, Porosity-Graded and Lightweight Lattice Structures Made by Additive Manufacturing," Finite Elem. Anal. Des., **138**, pp. 1–11.
[60] Panesar, A., Abdi, M., Hickman, D., and Ashcroft, I., 2018, "Strategies for Functionally Graded Lattice Structures Derived Using Topology Optimisation for Additive Manufacturing," Addit. Manuf., **19**, pp. 81–94.
[61] ASME, 2009, *Dimensioning and Tolerancing*, American Society of Mechanical Engineers, New York, Standard No. ASME Y14.5-2009.
[62] ISO, 2012, "Geometrical Product Specifications (GPS)—Geometrical Tolerancing—Tolerances of Form, Orientation, Location and Run-Out," International Organization for Standardization, Geneva, Switzerland, Standard No. ISO 1101:2012.
[63] Srinivasan, V., 1999, "A Geometrical Product Specification Language Based on a Classification of Symmetry Groups," Comput.-Aided Des., **31**(11), pp. 659–668.
[64] Walker, R. K., and Srinivasan, V., 1994, "Creation and Evolution of the ASME Y14. 5.1 M Standard," Manuf. Rev., **7**(1), pp. 16–23.
[65] ISO 10303-242, 2014, "Industrial Automation Systems and Integration—Product Data Representation and Exchange—Part 242: Application Protocol: Managed Model-Based 3D Engineering," International Organization for Standardization, Geneva, Switzerland, Standard No. ISO 10303-242.
[66] ASME, 2003, *American Society of Mechanical Engineers, Digital Product Definition Practices ASME Y14.41-2003*, American Society of Mechanical Engineers, New York.