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Challenges and Opportunities in Geometric Modeling of Complex Bio-Inspired Three-Dimensional Objects Designed for Additive Manufacturing

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Ever since its introduction over five decades ago, geometric solid modeling has been crucial for engineering design purposes and is used in engineering software packages such as computer-aided design (CAD), computer-aided manufacturing, computer-aided engineering, etc. Solid models produced by CAD software have been used to transfer geometric information from designers to manufacturers. Since the emergence of additive manufacturing (AM), a CAD file can also be directly uploaded to a three-dimensional (3D) printer and used for production. AM techniques allow manufacturing of complex geometric objects such as bio-inspired structures and lattice structures. These structures are shapes inspired by nature and periodical geometric shapes consisting of struts interconnecting in nodes. Both structures have unique properties such as significantly reduced weight. However, geometric modeling of such structures has significant challenges due to the inability of current techniques to handle their geometric complexity. This calls for a novel modeling method that would allow engineers to design complex geometric objects. This survey paper reviews geometric modeling methods of complex structures to support bio-inspired design created for AM which includes discussing reasoning behind bio-inspired design, limitations of current modeling approaches applied to bio-inspired structures, challenges encountered with geometric modeling, and opportunities that these challenges reveal. Based on the review, a need for a novel geometric modeling method for bio-inspired geometries produced by AM is identified. A framework for such a bio-inspired geometric modeling method is proposed as a part of this work. [DOI: 10.1115/1.4051720]

Keywords: computational geometry, geometric modeling, computer-aided design, design process, design representation, design visualization, product design, product development

1 Introduction

Ever since the early days of humankind, nature has been an immense source of inspiration when it comes to designing and inventing [1]. For example, more than 2000 years ago, people in Asia have noticed that some trees, such as *Picea abies* (commonly known as European spruce) illustrated in Fig. 1(*a*), have their branches shaped in a way that raindrops slide along them fast and rainfall water does not hold on them for long [3]. As there was need in preventing roof leakage, it is believed that this idea was adapted into the roof building process and can be traced up to pagoda roofs illustrated in Fig. 1(*b*), which are common in China, Korea, Japan, and other regions of Asia [4,5]. The intuition of the ancient people led them to the right solution, as this shape appeared to be a so-called brachistochrone curve—an optimal curve of fastest descent and thus does not let water to stay on roofs for long [6]. Figure 1(*c*) shows a plot of a brachistochrone curve and the specific time required to travel along it, as well as plots of a circular arc, a parabola, and a straight line for comparison. The history of design is full of other examples of bio-inspiration.

Another example is the glass sponge with complex hierarchical structures, which inspires some modern architectures in the world, such as the Swiss Re Tower in London, UK, and the Eiffel Tower in Paris, France, shown in Fig. 2(*b*) [7]. The intricate skeleton of glass sponge is shown in Fig. 2(*a*) [7]. The structure is strong and flexible even though it is made of fragile glass. The reason is that the glass sponge has complex hierarchical and lightweight structures from nanometer to macroscopic length scales, and they have been evolving to overcome the brittleness of the glass material, which helps it achieve lightweight combined with high strength [8].

This lightweight structure of the glass sponge also inspired engineering designs of tube-shaped and thin-walled structures such as the bio-inspired and honeycomb lightweight structures produced by additive manufacturing (AM) as shown in Fig. 2(*c*) [8]. These two structures have been tested through finite element modeling (FEM) analysis to compare the difference between their material properties under a certain compression condition. As seen in the compression-displacement curve of different structures shown in Fig. 2(*d*), the honeycomb structure was not able to provide good structural compression-bearing ability and low lightweight numbers compared to bio-inspired structure I and bio-inspired structure II structures, which indicates that the structure inspired from glass sponge performs better on the compression, bending, and torsion capacity. Therefore, this lightweight structure inspired by glass sponge can potentially be widely used in the industry sectors requiring low weight and providing high reliability, such as aerospace and automotive industries.

Another similar example would be the aircraft structure inspired by the honeycomb structure. The honeycomb illustrated in Fig. 3(*a*) is composed of hexagonal cellular structures which provides the most stable containment using the least amount of material [11]. A cross section of a rotor blade is presented in Fig. 3(*b*), which is composed of various composite materials to produce a lightweight

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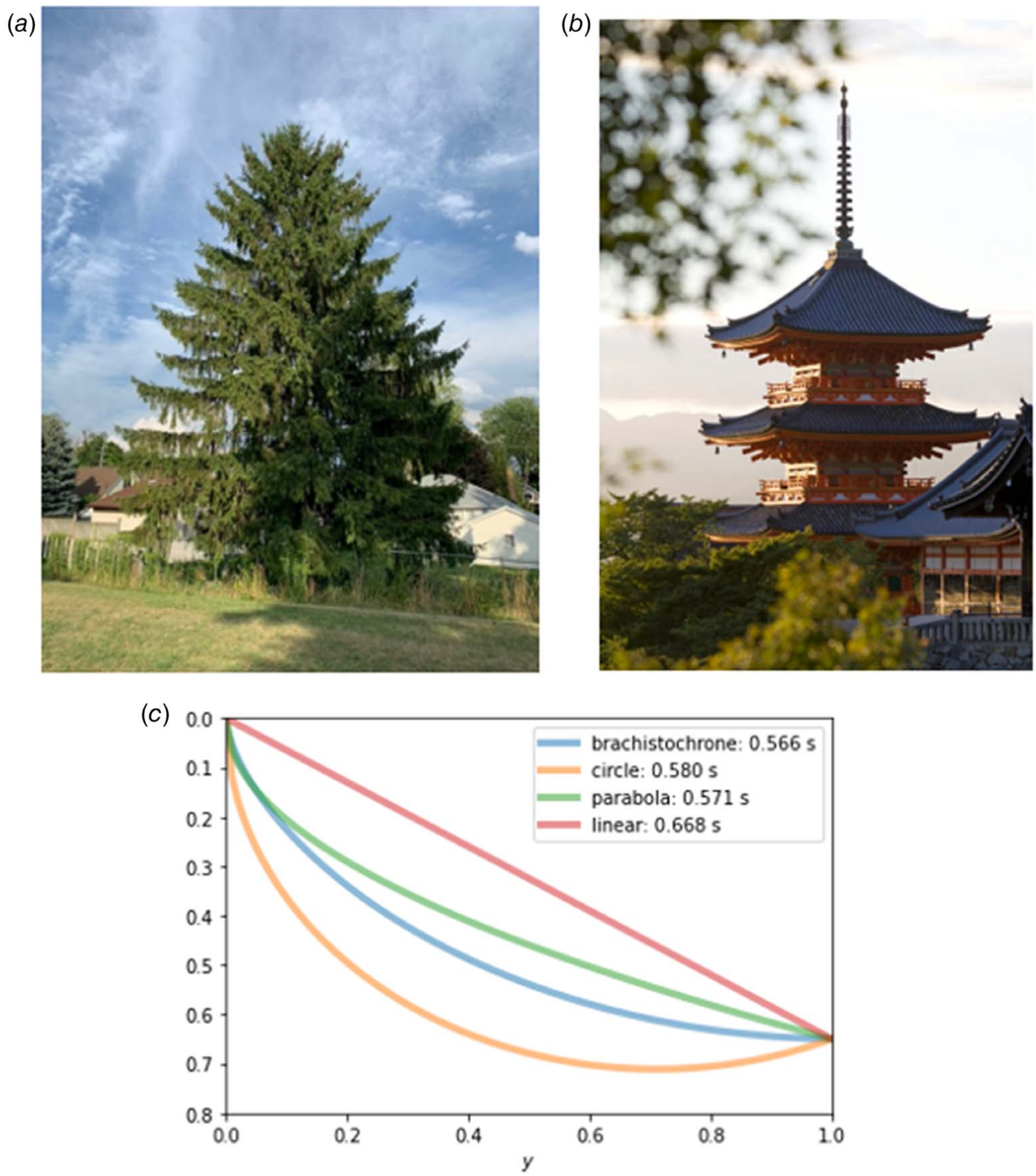

Fig. 1 (*a*) Branches of a conifer tree, (*b*) the roof of the Kiyomizu-dera temple in Kyoto, Japan [2], and (c) brachistochrone curve—an optimal curve of fastest descent—to prevent rainfall water from staying on them

and strong rotor blade. The rotor blade incorporates the honeycomb structure because the rotor blade should be strong enough to provide the lifting force for the helicopter along with the adjustments of the angles of its blades while being as light as possible [11]. The bonding of the "green" (environmentally friendly) Nomex honeycomb core and metal skin also allows the designer to form desirable shapes into blades which increase the performance in terms of beam strength [12].

Solid modeling has been intensively used by engineers and designers ever since the introduction of the first computer-aided design (CAD) software packages. While conventional geometric modeling has proved itself useful for engineering design, it began to fail in meeting the demands of AM techniques. In this work, AM is defined as "the process of joining materials to make objects from 3D model data, usually layer upon layer, as opposed to subtractive manufacturing methodologies" [13]. Existing CAD software packages and their geometric modeling kernels (GMKs) are not able to handle the significantly increased complexity bio-inspired design typically creates. They are also extremely challenged to model the complex structures that AM technology can easily fabricate such as heterogeneous lattice structures. In other words, the manufacturing capabilities develop faster than the geometric modeling capabilities required to support manufacturing. In this work, a geometric model is considered to be complex if (1) it is more difficult to model it with Boolean descriptive modeling rather that with parametric modeling or (2) it is not possible to support 30 frames-per-second (FPS) frame rate performance on an average computer used in engineering. In this work, a machine is considered average if it can provide 16 GB of random-access memory, 2 GB of disk space allocated on a solid-state drive, and a 64-bit central processing unit (CPU) with the clock signal frequency of 3.3 GHz. These system requirements are identified according to the recommended system requirements for SOLIDWORKS and RHINOCEROS 3D software packages which are extensively used for 3D modeling [14,15]. The threshold of 30 FPS is chosen as it is proven to be sufficient for convenient work and observations [16].

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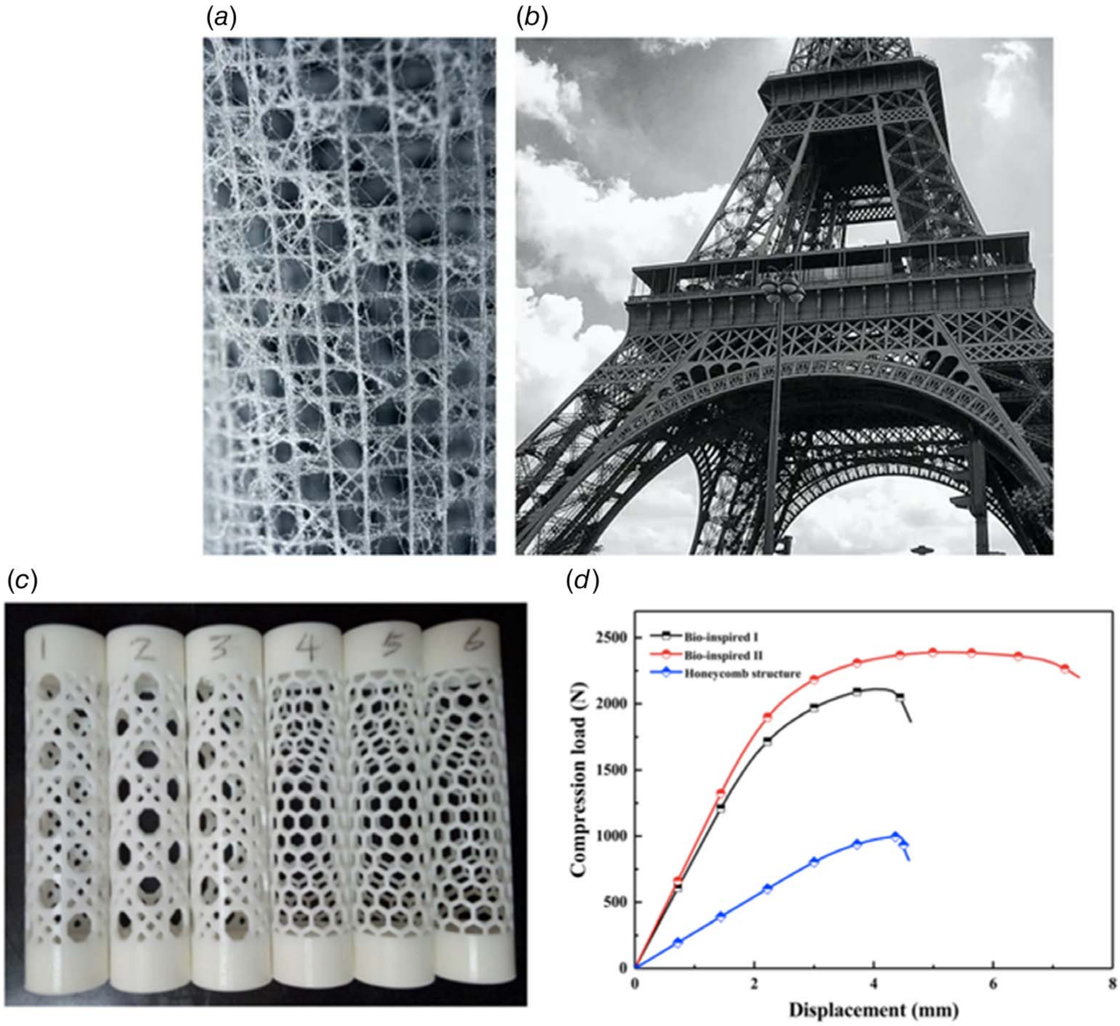

Fig. 2 (*a*) The intricate skeleton of glass sponge [7], (*b*) the Eiffel Tower in Paris [7] (Attribution-NoDerivatives 4.0 International (CC BY-ND 4.0)), (*c*) bio-inspired structure and honeycomb structures, and (*d*) the compression versus displacement curves for three bio-inspired structures. (Permission to reprint from IOP © 2020 [8].)

There are various applications of AM such as parts consolidations, weight reduction, functional customization, personalization, and esthetics [17,18]. Numerous research domains are utilized in AM: computational optimization, geometric modeling, behavioral simulation, material science, etc. [19,20]. Even though these domains utilize different software tools, methodologies, and approaches, they cannot be considered separately, e.g., computational optimization can be applied not only to weight reduction but to parts consolidation as well. The geometric modeling tool for AM needs to be able to support representation of these multi-domain simulations.

Geometric modeling in engineering is applied as early as at the conceptual design stage and the geometric model is used throughout

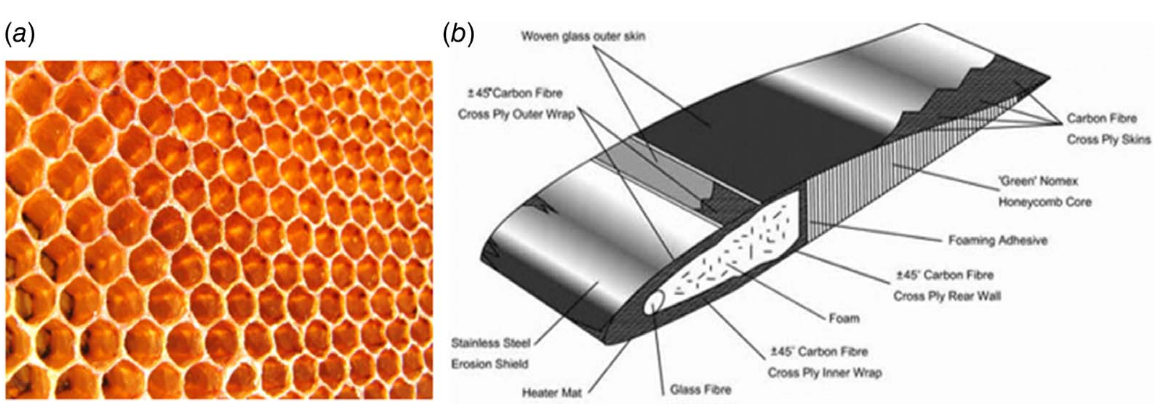

Fig. 3 (*a*) The honeycomb structure [9] and (*b*) the cross-sectional view of the composite rotor blade [10] (Attribution 2.0 Generic (CC BY 2.0) https://creativecommons.org/licenses/by/2.0/)

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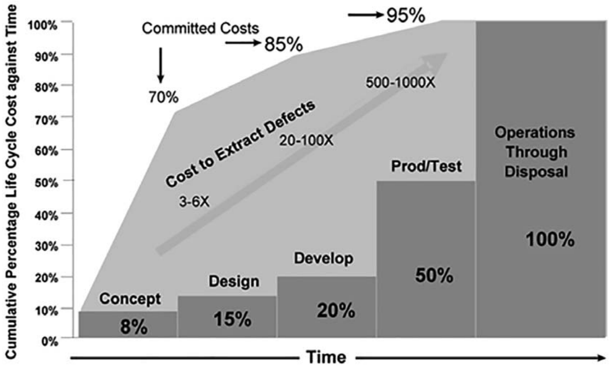

Fig. 4 Committed lifecycle cost versus time [21] (Permission to reprint from John Wiley and Sons © 2014)

the lifecycle of the product development. The conceptual stage of a product lifecycle is one of the most crucial ones. Costs committed to the initial—conceptual—design stage of the lifecycle are found to reach 70%, while only 8% are being spent during this stage as illustrated in Fig. 4 [21]. Thus, it is crucial to provide the initial stages of lifecycle with efficient geometric modeling tools.

Current geometric modeling techniques tend to fail in supporting AM due to an increased design freedom provided by AM technology which can support high geometric complexity of manufactured parts. Since solid modeling is based on classical topology and geometry, the higher the geometric complexity, the harder it is to model it [22], especially when the geometry is bio-inspired and does not follow the common design rules [23]. This complexity cannot be provided by explicit modeling mainly due to enormous amount of Boolean operations required to design a single geometrically complex part [24] such as lattice and bio-inspired structures seen in Fig. 2(*c*) [25,26]. It has been identified that there is no sufficient geometric modeling tool that would be able to represent complex heterogeneous lattice structures which form a research gap that is yet to be filled [27].

The rest of the paper is organized as follows. In Sec. 2, concepts and current state of the geometric modeling and computer graphics are reviewed. Similarities of geometry discretization techniques with natural processes are also identified; the design and geometric modeling methods of bio-inspired geometric objects are covered. Section 3 separately focuses on geometric modeling of lattice structures as they are identified to be a subset of bio-inspired structures but with unique features that make them distinguishable from the rest of bio-inspired structures. In Sec. 4, the discussion of this review is made and an architecture of a potential tool for complex geometric object design is proposed and discussed. In Sec. 5, future prospects are identified, and conclusions are made.

2 Current Status of Geometric Modeling of Bio-Inspired Complex Structures

Any engineering software that processes a 3D model, whether it is CAD, computer-aided manufacturing, or computer-aided engineering, has a GMK at its core with other tools supporting it. GMK is responsible for building numerical models of required geometries via mathematical methods [28]. Geometric modeling of complex bio-inspired structures requires a thorough review as it has significant challenges identified, mostly related to defining the bounding shape and computational optimization of a GMK [29,30]. The domains of mathematics behind every GMK include linear algebra, topology, mathematical logics, graph theory, and more [31,32]. In other words, a GMK is mathematics turned into code so that the geometric information can be viewed on a screen. The algorithms utilizing these domains of mathematics work best when they are developed using programing techniques which can

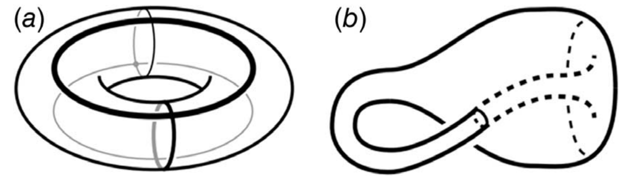

Fig. 5 Two closed two-manifolds: (*a*) an oriented two-manifold of genus 1 (torus) $T = M_1^2$ and (*b*) a non-oriented two-manifold of genus 2 (Klein's bottle) N_2^2 [39]

provide high-level functionality [33,34]. A GMK is usually developed by a large team of software developers and mathematicians for several years and thus it is very challenging to develop a GMK by a small group of people [35].

A geometric object describes the form of the modeled object [36]. Geometric objects include curves, surfaces, bodies, as well as topological objects that describe geometric properties that do not depend on quantitative features and describe permanently interconnected points in 3D space [37]. There are 2D and 3D geometric objects. Two-dimensional objects are used to work in definition areas of surface parameters, as well to work with planes of local 3D coordinate systems [31]. In this work, a 3D object is considered to be defined according to the functional representation (F-rep) methodology, i.e., by a real-valued function $F(X)$ where $X=(x, y, z) \subset \mathbb{R}^3$ is the design space, such that $F(X) \leq 0$ is the object itself with $F(X) = 0$ being the object's surface and $F(X) > 0$ is the rest of the design space [38]. To visualize these objects, most of the existing CAD software packages use GMKs for handling geometric information and making it available for user to see, which work together with parametric modeling kernels that support Boolean operations, constraints, etc. [31].

Topologically, the surface of a geometrically complicated part such as a lattice structure is a closed oriented two-manifold M_g^2 of a significantly large non-zero genus ($g \gg 0$). In topology, a closed two-manifold is a connected surface that exists in three dimensions. They are oriented if there is no path from one side of a surface to another, as seen in Fig. 5. In this work, only orientable two-manifolds are taken into account as only a solid body bounded by an orientable two-manifold without intersections is manufacturable. A single simple unit grid has genus 5, meaning that it has many curvatures and detail on their micro-scale [40].

Polygonal meshes begin to fail when complex geometric objects are modeled with them such as bio-inspired structures and heterogeneous lattice structures. One of the most popular non-proprietary CAD file formats—stereolithography (STL)—utilizes polygonal representation [41]. In polygon surface mesh, number of finite elements rises exponentially with model complexity and severely impacts modeling of complex shapes by making it way too computationally expensive as seen in Fig. 6 [43].

Note that mindlessly increasing the number of nodes stops showing any improvement at some point and that critical non-plane areas normally require smaller element size [42]. Moreover, smaller finite elements not only increase calculation time but also introduce errors in geometry representation as seen in Fig. 7. This calls for approaches different from the ones used in most CAD software packages.

The so-called influencing points of increased complexity of the mesh require mesh edges to be orthogonal to the surface boundary for increased performance and decreased error-proneness [45]. Note that in Fig. 6, the mesh becomes denser when nearing the influencing points, in this case located near the surface boundaries and at non-plane surfaces. This requires extra calculations made which slows the mesh generation and the modeling process corresponding to it. The interpolation based on radial basis functions (RBFs) attempts to improve the performance of these operations significantly [46]. However, this approach was initially designed for 2D mesh generation and still requires certain improvements to be widely used in 3D. For example, it has been found that RBF

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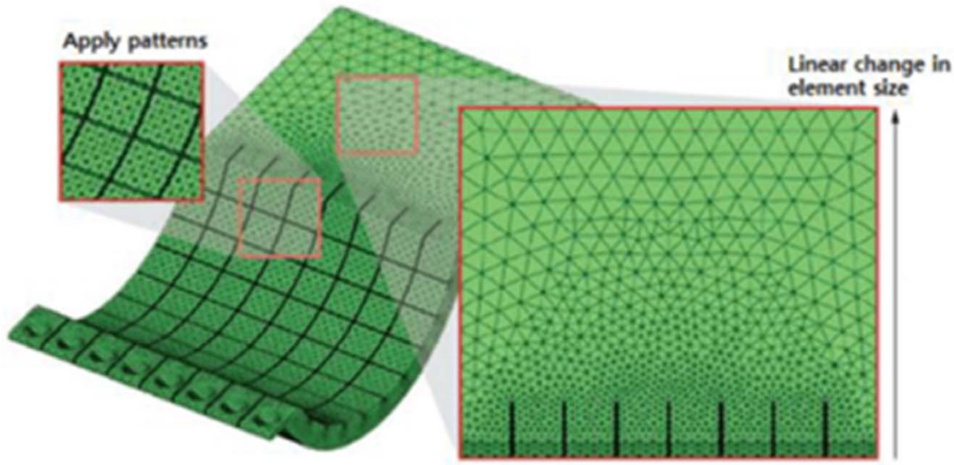

Fig. 6 Model complexity affecting computation expense of rendering a surface mesh [42] (Reprinted with permission from Cyprien Rusu)

interpolation may fail in case it is applied to a closed oriented surface such as, for example, a full cylindrical surface. The reason for that is the failure to detect what are influencing points in case of a completely symmetrical and closed surface as every point is influential in this case. The possible solution for it lies in applying hybrid methods that introduce parallelization to the process, but even then it requires top tier CPU capabilities [47].

Mesh modeling assumes that a solid model is defined by tiny finite elements (often triangular) each of which can be defined by vertices, as well as the position and orientation of the element in the design space. For example, bellow is an example of a triangular finite element defined in an American Standard Code for Information Interchange (ASCII) stereolithography (STL) file by its normal vector and vertices:

```
facet normal 0.95105690250522623
 0.30901580574003779 -0
 outer loop
  vertex 14.842915534973145
  11.243449211120605 -5
  vertex 14.648882865905762
  11.840622901916504 -5
  vertex 14.648882865905762
  11.840622901916504 0
 endloop
endfacet
```

Boundary representation (B-rep) techniques have been evolving rapidly and incorporated into major GMKs such as parasolid and Open CASCADE. B-rep allows modeling of solids made by revolution, extrusion, chamfering, and other operations with solids common in modern CAD in addition to Boolean operations used prior to B-rep [48]. For example, a torus in B-rep can be defined as a circle ρ given by

$$\rho^2 - 2\rho R\cos(\theta) + R^2 = r^2 \quad (1)$$

where (ρ, θ) are the polar coordinates (polar coordinates are favorable in representation of circles and curves in B-rep due to decreased computation time [49]), r is the radius of the circle (and of the torus tube), and R is the distance between the origin and the center of the circle (and between the center of the torus and the center of its tube). This circle is then revolved around the z axis of the design space.

B-rep suffers from the same inability to model highly complicated geometries such as the ones in Fig. 2 for example. The reason for this is a lack of parametrization and numerous operations needed to achieve modeling of even a simple homogeneous lattice. The overall performance of B-rep methods can be improved by, for example, hybrid B-rep methods [50] and optimizing boundary spline (B-spline) functions [51,52]. However, this optimization is still limited by the number of operations and efforts needed to model geometrically complex structures even if functions are getting simpler, there are still way too many of them in complex structures. Moreover, the surface-to-volume ratios (SVRs) of the multi-scale and lattice structures can be thousands of times larger than the CAD models encountered in conventional design, which poses big issue for the modeling tools based on B-rep.

Non-uniform rational basis splines (NURBS) and their extension to surface modeling were introduced to mitigate difficulties

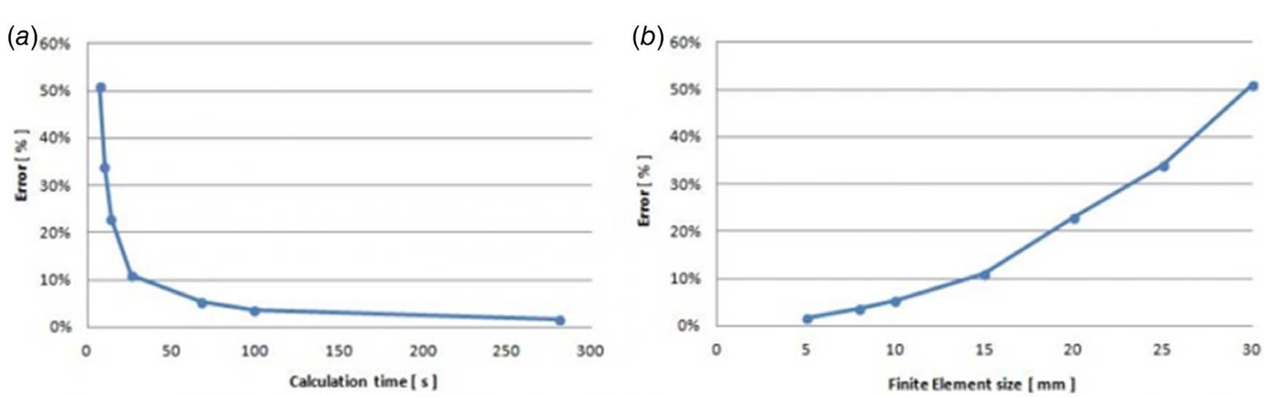

Fig. 7 Dependency of errors of geometric mesh representation: (*a*) on calculation time and (*b*) on finite element size [44] (Reprinted with permission of Lukasz Skotny)

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associated with modeling of complex structures and are used widely in B-rep [53]. NURBS surfaces and their trimming allow interpolation of the desired shape by points with simplicity. Initial Graphics Exchange Specification (IGES) and Standard for the Exchange of Product (STEP) are popular CAD file formats utilizing NURBS. However, trimming a NURBS surface $S(u(t), v(t))$ with a trimming curve $C(t)$ is not always possible, as it is not always possible to retrieve the knot vectors $u(t)$ and $v(t)$ for every parameter t [54]. Moreover, attempting to define an enormous amount of completely different NURBS surfaces for geometrically complex shapes makes the design process too tedious for a designer.

While B-rep does not operate with meshes, the mesh representation is still used for representing and rendering 3D models on screen. For example, even when a circle is defined in the design space, it still looks like a polygon with a number of vertices enough to be seen as a circle. Therefore, a certain conversion from B-rep to mesh is required to allow rendering of the model. This process is straightforward and has been extensively discussed in the literature [55,56]. Spline-based B-rep is precise enough for conventional engineering design. However, as AM started to allow a higher level of design freedom, more complicated shapes became manufacturable. There appears to be a trade-off between having a higher quality of geometry and having a more complex geometry. Note that the inverse problem is not that straightforward and encounters issues often associated with this type of problems, which are mostly related to a necessity to develop a feature recognition algorithm [57,58]. There are techniques that allow rendering of shapes with curvature explicitly, and the development of these techniques significantly contributed toward research on F-rep since these shapes often require an explicit function that controls its curves [59,60].

The lack of an appropriate tool to model bio-inspired structures makes it challenging to design them as well since designers often have difficulties to design micro-structures that are mimicked from animals or plants. Designers often try to replicate the actual structure/surface feature to obtain the maximum desired functionality during the conceptual development phase when sketching out their ideas [61]. For example, protective surfaces and structures that are used to protect dental implants when they are subjected to chemical etching process to enhance osseointegration process are hard to design [62,63]. Moreover, printing the desired surfaces with the desired material is often a challenge as well. Keeping these challenges in mind, designers often try to simplify the design by re-defining the concept and seek inspiration from other animals or organisms [64,65]. For instance, consider a conceptual design of a gecko inspired surface sketched in Fig. 8 as an example of a design that is required to go through functional and geometrical changes to be able to be modeled and manufactured.

The aim of this design is to provide a sticky surface and protect the upper part of implant when the dental implant is processed to chemical etching. Designing the setae (micro-hair) of the gecko's feet is a challenging task and to print them using a desired material is another challenge [66]. Due to these challenges, geometric modeling, and fabrication techniques, a transition to the design inspired by ant's claw serving the same function was made with two potential conceptual designs sketched in Fig. 9(*a*). Figure 9(*b*) provides a more detailed version of the conceptual design and Fig. 10 provides the final CAD design which carries significantly less resemblance with the initial bio-inspired conceptual design. This is mainly due to the modeling and manufacturing issues arising from the geometric complexity of the initial conceptual design.

This is only one example of a bio-inspired design affected by limitations of geometric modeling and manufacturing. There are many more including bio-inspired lattice structures resulted from topology optimization which often require top tier graphics processing capabilities and highly capable geometric modeling tool [67].

Normally, in bio-inspired design, the focus is made on a fixed set of functions, e.g., water resistance and/or increased stiffness [68]. However, organisms in nature often combine much more than just several functions. Moreover, the functions of living organisms are weighted differently. As an example, consider a camel in a

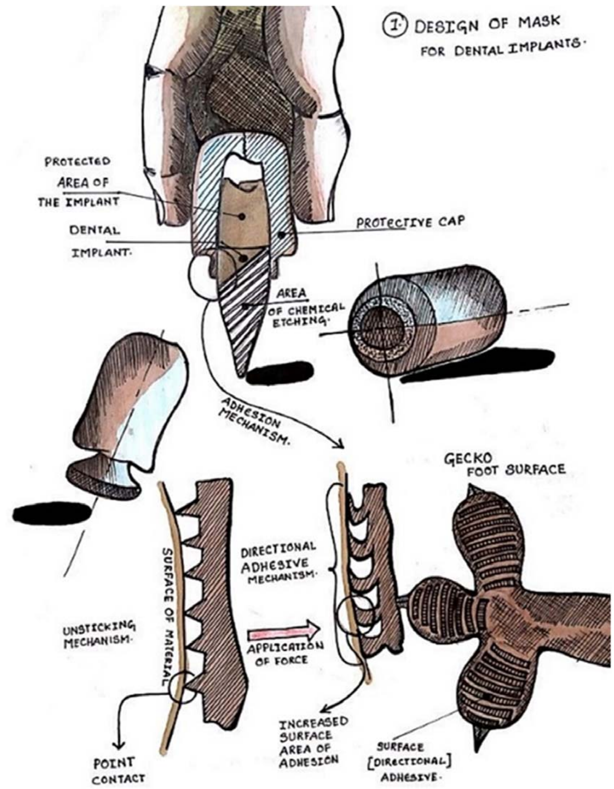

Fig. 8 The conceptual sketch of a gecko inspired surface that sticks to the implant surface [61]

desert: it does not focus that much on finding water (this task has not 0% weight for it, but it is not 100% either). Instead, it prioritizes more on storing the water in its hump and spending it carefully afterwards (with a weight much more than of just finding water) [69]. In the modern bio-inspired design, the weights are essentially set to 0% or 100%, so in multi-functional structures, the focus is made on some set functions and the other possible functions are neglected, even though several functions might actually solve the same problem: the camel solves his hydration problem as in the example above, but many other desert animals solve their hydration problem by actively searching for water at night (they get it from plants, mostly). Therefore, an investigation on how bio-inspired functions can be explicitly identified would be of great use, as they are required as a crucial input for function-based bio-inspired heterogeneous lattice structures modeling [70,71].

To further explain the current status of geometric modeling methods for bio-inspired complex structures, the rest of Sec. 2 is organized as follows. In Sec. 2.1, discretization occurring in nature is covered and similarities with discretization in geometric modeling and computer graphics are identified. Section 2.2 reviews volumetric modeling techniques applied to complex geometric structures. Section 2.3 focuses on the multi-scale aspect of geometric modeling and challenges encountered in this aspect.

2.1 Discretization in Nature. Discretization in geometric modeling and computer graphics is a topic of great interest as it allows control over the mesh size and density, thus directly controlling quality and complexity of the model. Similarly, discretization has a crucial role in the structure of living organisms as they are made of small living building blocks—cells. The algorithms of growth and development of living creatures form a prosperous research area of bio-inspired design. Moreover, some biological research involves geometric modeling of complex structures. For example, bio-inspired computational models and algorithms for simulating of 3D multicellular tissue growth form a prospective

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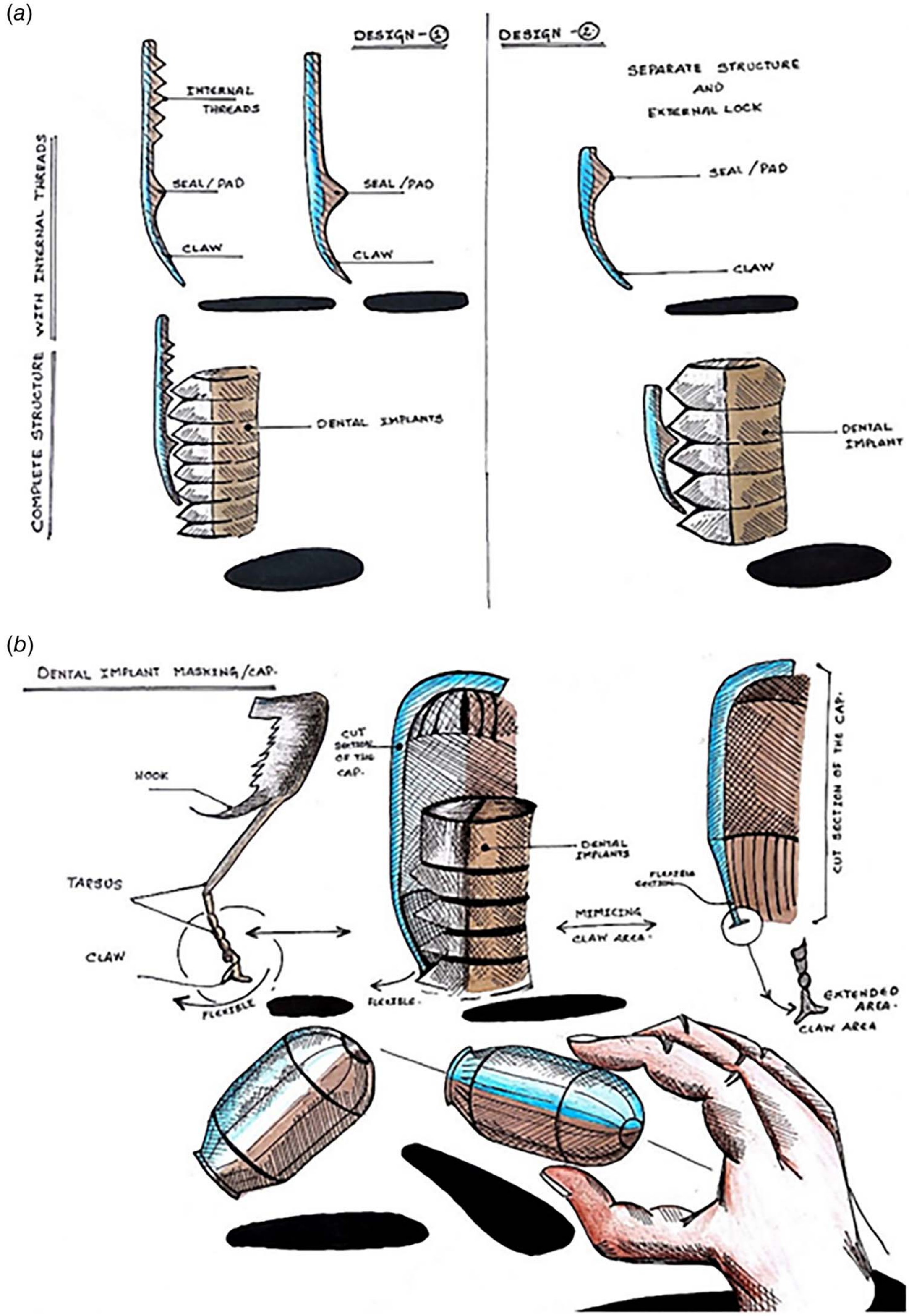

Fig. 9 **(*a*) Two conceptual designs inspired by an insect claw (ant's claw) and (*b*) the conceptual design of the dental implant masking/cap [61]**

research direction, which, however, lacks an appropriate geometric modeling tool [72].

Some recent research begins to dive into bio-inspiration when modeling or simulating complex bio-inspired structures. Dimas and Buehler [25] provided a novel modeling technique for bio-inspired composites which considers only a 2D cross section of a composite for modeling and is mainly focused on performing simulations [25]. Fantini et al. developed a method to design bio-inspired structures based on Voronoi lattices [73].

When considering discretization in nature, the first thing to review is the structure of living organisms such as humans. The evolution process made the simplest organisms on Earth converge

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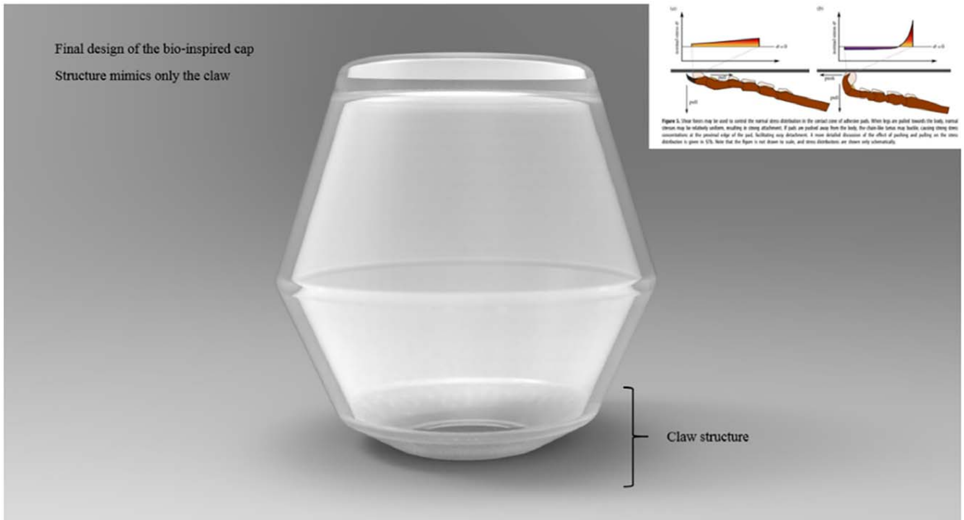

Fig. 10 Render of a protective cap inspired by an insect claw (ant's claw) [61]

to complicated and robust species. It resulted in developing optimal shapes and structures that are parts of living organisms developed in billions of years, e.g., scutoid cells which are 3D solids bounded by two polygons lying in parallel surfaces (not necessarily planar) and with vertices interconnected either by curves or by Y-shaped connection as seen in Fig. 11 [74]. Thus, there are yet any bio-inspired algorithm or a technique to adapt from nature into geometric modeling.

Surface mesh modeling resembles discretization similar to the way human skin consists of skin cell but does not model the interior. Voxels normally discretize a design space into cubes, while cell geometry is not necessarily cubic [30]. Note that the possibility of using non-cubic voxels was investigated and tested with various voxel shapes such as body-centred cubic (BCC) and face-centred cubic (FCC) (BCC is similar to a truncated octahedron and FCC is similar to a rhombic dodecahedron) [75]. The results of applying non-cubic were not encouraging enough as non-cubic grids appear to be more sparse than cubic grids that provide the most information about the structure. Currently, the sparse voxel octree technique allows modeling with voxels of different sizes [76]. However, the cubic shape of voxels remains the same which does not allow the variety of shapes that is present in nature and cubic voxels introduce anisotropy which depends on the orientation of cubic grid [75].

Considering that the idea of bio-inspired structures comes from nature, it is important to be able to model the variety of shapes. Moreover, as will be covered in Secs. 2.2 and 2.3, both mesh modeling and voxel modeling have certain computational and accuracy-related disadvantages when applied to complex geometric objects. Thus, for bio-inspired geometric modeling, other bio-inspired geometry classes must be considered [77].

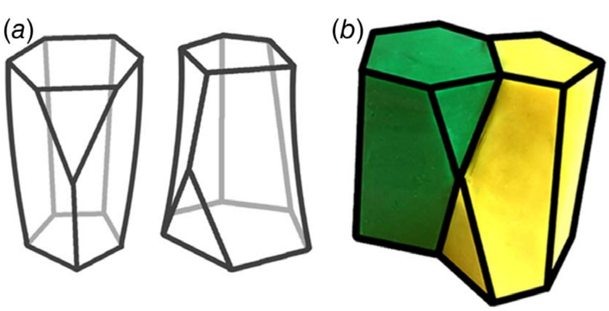

Fig. 11 Two scutoids (*a*) shown transparent separately and (*b*) shown opaque and transitioning one into another [74] (Permission to reprint from Springer Nature © 2018)

In nature, there are numerous shapes and sizes of cells that together emerge into a living body. These parameters are greatly defined by the cell division process, which in turn is defined by genetics and external conditions. Thus, it is required to consider the cell division process in more detail.

There are numerous rules that are followed in cell division. One of them is the long axis rule (LAR) that has been observed in nature and it defines the cleavage plane as the plane perpendicular to the longest axis passing though the center of mass of cell [78]. Considering that a geometric model is not assigned any material, the centroid could be taken instead of the center of mass.

However, there are cases when the LAR is not satisfied, which readily indicates that the LAR is not followed in nature all the time and that there are other algorithms that define the shape and position of cells. The other rule is the SVR minimization [79]. The smaller the SVR is, the less the cell is exposed to commonly unfriendly external environment [80]. The most optimal shape from this point of view is a sphere which is not always feasible due to external conditions such as neighboring cells and other geometric constraints. However, in some cases, the SVR tends to be maximized, e.g., trees having a larger leaf area receive more sunlight and carbon dioxide and are able to survive better [81].

This already suggests that if there is a bio-inspired geometry discretization technique, it cannot be a single algorithm but rather a combination of several algorithms with a tuned trade-off method. The way the trade-off between different cell division processes occurs in nature is still an open question in cellular biology [78]. Note that even though only two biological rules are presented in this review, in nature there are many more and they continue to be constantly discovered [82].

In one of the recent works, a method that takes the above mentioned two rules is described in depth in an approach using volumetric cells [40]. The flowchart in Fig. 12 illustrates the approach. In this algorithm,

$$C := \bigcup_{i=1}^{n} \left(V_i / \bigcup_{j=1}^{m} \partial V_i^j \right) \subset \mathbb{R}^3 \quad (2)$$

is the whole structure which is subdivided into numerous volumetric cells V_i^j in k steps until the maximum number of steps $k_{\max}$ is reached. However, the implementation is demonstrated for 2D cases which are insufficient for supporting complex 3D geometries. Moreover, the trade-off algorithm between the two rules has not

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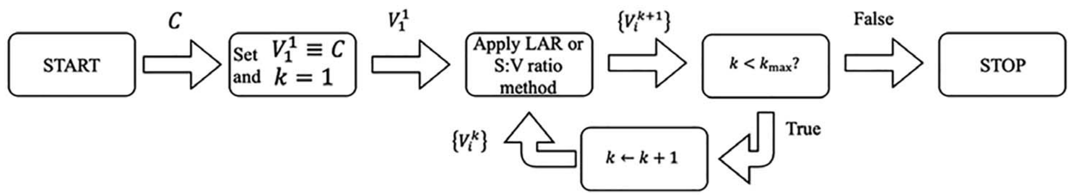

Fig. 12 The bio-inspired geometric modeling algorithm based on the LAR and the SVR minimization rule observed in nature [40]

been developed and the choice of the method for each iteration is performed manually.

Indeed, taking a close-up look at epidermis, the upper layer of human skin shown in Fig. 13 reveals that it looks similar to Voronoi tessellation and is extensively used for modeling human skin, as well as for FEM mesh modeling seen in Fig. 6 [83,84].

Altair SimSolid uses specific algorithms for recognizing features within a model, e.g., planes, bolts, screws, etc., thus describing the model with a complex but single mathematical equation, which allows avoiding meshing for simulations [85]. In case of, for example, detecting a bolt, it works by detecting a hexagonal head on top of a cylinder and thus assuming that this solid body is a bolt. However, this feature recognition of Altair SimSolid appears to be pre-defined, and it fails to detect screw that do not have hexagonal heads, for example. Some feature recognition algorithms, including the bio-inspired ones, might find application in discretization in geometric modeling [86]. Moreover, a single mathematical equation might as well be non-computable due to its complexity.

2.2 Volumetric Modeling. Challenges with surface mesh modeling force developing of tools utilizing voxel modeling for designing complex geometric structures [87]. Voxel modeling has been used for eliminating high-frequency details of the object ever since the introduction of voxels [88,89] which is essential for modeling complex structures such as bio-inspired ones. Moreover, voxels have an advantage in terms of downsampling and acquisition of real-world data [76]. Moreover, there is no need in voxels smaller than 3D printer resolution as they would not be manufacturable [90]. Voxelized models support the same Boolean operations as the mesh models [87]. A significant advantage of voxel modeling for AM lies in straightforward machine learning applications, such as prediction of model printability [91].

In voxel modeling, voxels normally have a cubic shape [75] with some non-cubic approximations such as the ones produced by the marching cubes algorithm [92,93]. Having the same element tessellated in the design space results in inaccurate representation of curvatures in case of having not enough voxel density, while having a significantly high voxel density results in high computational expenses. Applying the level-set method (LSM) allows considering a voxelized 3D design space as a set of 2D layers which improves the computational complexity from $O(n^3)$ to still rather complicated $O(n^2)$ [94].

Applying voxelization as it is without any additional optimization is still computationally expensive [95]. One of the most popular voxel-based simplification methods involves using sparse voxel octrees which are based on generating multi-scale voxels which could be visible or invisible depending on the resolution, size of the screen, and point of view [76]. This approach applied to large voxel models can result in up to six times increased efficiency [96].

Another volumetric modeling approach is the finite volume method which generates volumetric mesh similar to surface mesh but with the whole solid body discretized rather than just its surface, i.e., the body is subdivided into polyhedrons, not polygons [97]. However, this approach has disadvantages similar to surface mesh: computation of curvatures is nontrivial due to their geometric complexity and the computational expenses rise exponentially with complexity.

Note that unit elements in every existing geometry discretization technique are always convex, whether they be finite elements or voxels. Convex unit elements require less computation, but it is required to have more unit elements to model strongly non-convex shapes such as bio-inspired lattice structures. This implies the need in a proper meshing algorithm that takes convexity and curvature into account and affects quality of meshed models, especially the ones requiring multi-scale modeling [98]. Thus, there is critical need to identify whether non-convex unit elements such as the one sketched in Fig. 14 could be used for geometric modeling of non-convex geometries. Note that combining two convex finite elements into one non-convex would result in a computationally less efficient finite element but would also reduce the total number of finite elements, introducing a trade-off between these two parameters. It should be investigated in more detail whether such a trade-off of computational efficiency of rendering separate unit elements for a lesser amount of unit elements is beneficial for the whole model rendering efficiency.

Volumetric representation (V-rep) modeling is another volumetric modeling technique that has been introduced recently and that has brought attention from the engineering community [99].

Fig. 13 A close-up on human skin

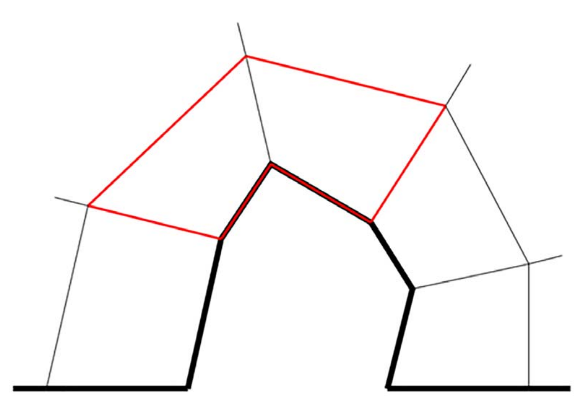

Fig. 14 A mesh of non-convex geometry

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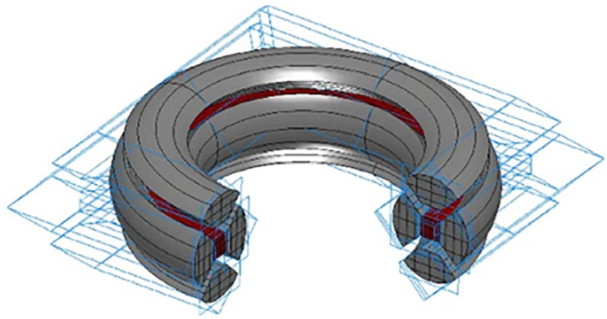

Fig. 15 A torus is constructed using five solids of revolution in V-rep modeling [100] (Creative Commons Attribution 4.0 International (CC BY 4.0))

It utilizes elliptic partial differential equations and modifies the design space to have a variety of unit volumes that handles extreme geometric complexity. The approach is superior to B-rep modeling in terms of geometric complexity handling [100] and can be adapted for simulation purposes easier [101]. However, it suffers from a similar issue with B-rep: while in B-rep two surfaces collide by an edge or a group of edges, and in V-rep two volumes collide by a surface or a group of surfaces, since surfaces are in general more computationally costly than edges [102]. This can dramatically increase the computational expenses in cases with a large amount of unit volumes. There is evidence that the hybrid B-rep approach discussed previously in Sec. 2 can be applied to V-rep to increase its performance as well [50]. The torus that was used as an example in the previous section can be represented in V-rep as a union of five solids of revolution as seen in Fig. 15. Note that the "core" of the torus is required to be a separate solid to avoid convergence of other four solids to zero.

Volumetric modeling with iso-geometric finite elements utilizes cubic finite elements that are transformed to fit the desired model better by, for example, moving vertices of the default cube to new positions as illustrated in Fig. 16(*a*). The resulting model consists of numerous iso-geometric finite elements as shown in Fig. 16(*b*). However, this method inherits drawbacks of both polygonal-based modeling (e.g., having irregularities at regions with high curvature) and voxel modeling (even though the variety of shapes is larger than having only one type of voxel, the finite elements are still limited to having six faces).

The described drawbacks of voxel modeling suggest modifying volumetric modeling techniques to better fit the rising demand for a geometric modeling approach that could support more complicated geometry. Using non-cuboid voxels which often find their use in computer graphics rather than in geometric CAD modeling improves the performance but introduces significant distortion to the model they are applied to. However, there is evidence that using a variety of unit volumes in a single model can dramatically improve both performance and quality of the model. The IRIT modeling environment does this by allowing modeling of so-called VModels with non-conventional unit volumes which allows storing 3D data in much smaller sized IRT files which are native to the IRIT modeling environment [104].

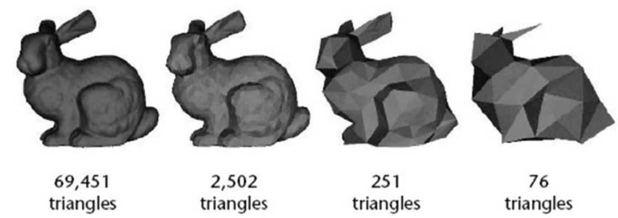

Fig. 17 Reducing the complexity of a 3D model by decreasing its level of detail that directly corresponds to the number of polygons required to render the model [108] (Permission to reprint from Elsevier © 2002)

Similar results have been shown in the work that introduces bio-inspired 2D cells which can potentially be brought to 3D as volumetric cells [40]. This bio-inspiration is based on natural cell division process—every living being is made of living cells and there is normally a huge variety of shapes of different cells. Since nature has been optimizing these shapes through billions of years of evolution, this suggests that there could be an optimal bio-inspired geometric modeling approach. The method is described in more detail in Sec. 2.1 as the algorithmic base for it is based on nature and is 2D at this stage.

2.3 Multi-Scale Modeling. Currently there are many challenges present in the area of geometric modeling of complex structures [105,106]. One of the most crucial challenges when it comes to modeling of bio-inspired structures is multi-scale modeling support, which enables the delivery of sufficient and accurate visual information from meso- and macro-scales [107].

In geometric modeling, the concept of level of detail (LoD) is applied widely to reduce computational cost, which essentially reduces model's complexity by decreasing the amount of details and vise-versa. Figure 17 shows an example of how model complexity changes when the number of polygons in a surface mesh model of the Stanford bunny decreases. Note that the higher of the LoD, the more details are rendered.

Normally, LoD is manually or automatically associated with CAD features as illustrated in Fig. 18 [109]. However, bio-inspired structures are normally designed using parametric modeling techniques and not explicit. This results in ambiguity in choosing what could be considered a feature corresponding to each particular LoD.

Interestingly, there are other areas of research, different from bio-inspired structure modeling, that are forced to deal with LoD ambiguity. For example, in geoinformatics, it is required to consider a 3D scan of a large archaeological site such as the one of the Maya civilization from various LoDs, starting with a whole Maya city (LoD0) and ending with ornament details of a column in one of the buildings (LoD3) [110]. A multi-scale geometric modeling approach could also solve problems in medicine [72,107], e.g., a need for a geometric modeling approach for modeling a human heart both as a whole and in details (as even small defects are crucial) is identified [111].

For voxel modeling, the depth of rendering (which is required for fully adaptive multi-scale voxel modeling [112]) is assumed to be

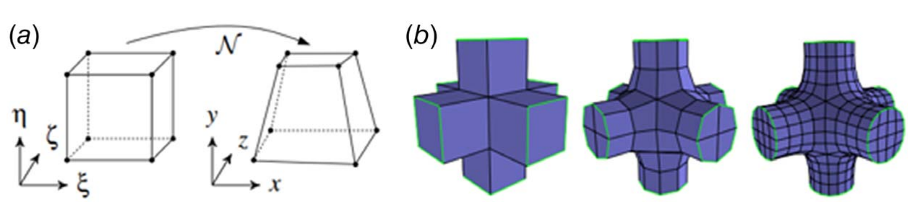

Fig. 16 (*a*) A transformation $\mathcal{N}$ of a cubic finite element to a new shape and (*b*) a geometric modeling of a structure with different stages of refinement with iso-geometric finite elements [103] (Permission to reprint from John Wiley and Sons © 2010)

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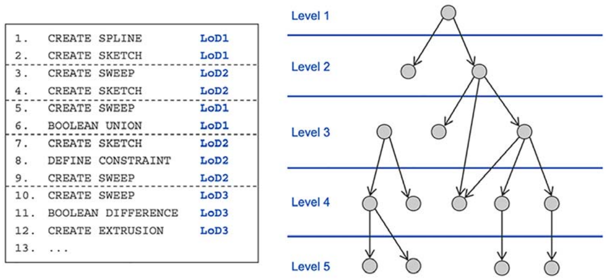

Fig. 18 Levels of detail associated with Boolean operations in CAD [109] (Permission to reprint from John Wiley and Sons © 2014)

given and the voxelization algorithm is not adaptive, which makes multi-scale modeling challenging with voxels.

Current voxel iteration methods are not adaptive due to difficulties in clustering of voxels [113,114]. This forms a research gap by having a lack of an appropriate adaptive voxelization algorithm for geometric modeling, i.e., automatic changing of voxel size depending on the distance to the user according to LoD: current approaches are unable to represent crucial features of a part on a larger scale with voxels [115] and become sufficiently slow on a smaller scale [95]. Thus, it is not clear which features should be associated with LoDs, how to recognize and classify the features, and which voxel size is sufficient to represent a feature.

3 Review of Geometric Modeling Methods for Heterogeneous Lattice Structures

A lattice structure is defined as "an architecture formed by an array of spatial periodic unit cells with edges and faces" [116]. Lattice structures are considered as a subset of bio-inspired structures in this work, as the idea of lattice structures comes from nature initially: they take their inspiration from hexagonal bee honeycombs, spider webs, internal sponge-like bone structure, etc. Lattice structures have been challenging to manufacture due to their complexity until the introduction of AM [117]. They provide an optimal performance-to-weight ratio and other unique properties that do not emerge from conventionally manufactured parts, e.g., gradual elasticity of the structure, water absorption or resistance, etc. [118–120]. They can be classified into homogeneous and heterogeneous as sketched in Fig. 19. In homogeneous lattice, the thickness of struts or nodes inside it stays the same over the entire structure, while these parameters vary in heterogeneous lattice [121]. Heterogeneous lattice structures appear often in the design of structures with optimized geometry such as the ones produced with topology optimization [122]. Note that while having heterogenous materials within the same structure is a popular topic of interest [123], this work focuses on geometric issues only and takes only geometrical heterogeneity into the account.

The concept of lattice structures is bio-inspired, but some lattice structures are bio-inspired in a way that they possess special shapes and properties of living organisms more vividly [124–126]. The majority of lattice structures in nature are heterogeneous as this enables to sustain more complex geometric shapes and a larger variety of them [127,128]. Mostly bio-inspired lattice structures

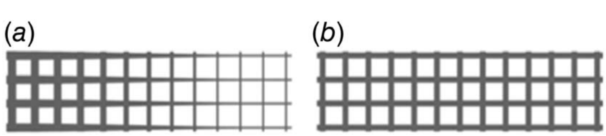

Fig. 19 (*a*) A heterogeneous structure and (*b*) a homogeneous lattice structure [121]

are utilized in very specific use-cases, e.g., when it is required to provide properties that are unique to certain biological species [129–131]. For instance, An and Fan [124] provide an example of a heterogeneous sponge-like lattice structure that makes a part ultra-lightweight while maintaining its strength and energy absorption, showing an example of a completely heterogeneous non-periodic lattice structure. Geometric modeling of homogeneous lattice structures has been extensively covered in the literature [132–134], which is not the case for heterogeneous lattice modeling. With this in mind, this section specifically dedicated to discussing geometric modeling of *heterogeneous* lattice structures, covering their semi-periodic nature which introduces both challenges and opportunities. Such complex geometric objects like heterogeneous lattice structures have various parts that require different modeling techniques combined in the modeling tool as the geometry consists of features of different sizes and shapes, such as nodes and struts in lattice structures [135,136]. The issues with modeling of heterogeneous lattice structures often result in substitution of their models with homogenized versions or with 2D cross-sectional analog of them, with both of which being incapable of providing accurate information about a heterogeneous object [121,137].

Many of the issues and challenges arising in geometric modeling of lattice structures were analyzed in one of the previous works, including the application of LoD, as well as polygon mesh and voxelization algorithms [123]. This work, however, extends the previous review of geometric modeling of lattice structures, as the scope of this review lies also in identification of possible venues for the development of a tool for modeling of complex geometric structures, including heterogeneous lattice structures.

3.1 Challenges in Geometric Modeling of Heterogeneous Lattice Structures. Using conventional CAD for designing lattice structures has its own flaws which have been extensively reviewed in the literature [123,138]. The main issue with designing heterogeneous lattice structures with descriptive CAD systems is inconvenience and difficulty in describing even a homogeneous lattice with just Boolean operations, i.e., it is not trivial to associate some lattice feature with a set of Boolean operations, similar to associating LoDs to CAD features covered in Sec. 3 [139]. Designing heterogeneous lattice structures is even more challenging.

Using non-standard modeling solutions has proven itself useful for generation of homogeneous lattice structures with parametric design tools such as the Intralattice plugin for RHINOCEROS 3D, but they are still incapable of modeling heterogeneous lattice structures [140,141]. Moreover, heterogeneous lattice generation and visualization can still be slow when performed on an average machine, as mentioned in Sec. 1, mostly because of the complexity handling issue of the polygon surface modeling. Figure 20 illustrates several lattice topologies applied to the same design space using Intralattice. Note that these topologies are quasi-homogeneous: there is a pattern but in polar coordinates and not in Cartesian.

A similar tool which also works as a plugin for RHINOCEROS 3D is Crystallon distributed by the General Public Licence 3.0 (GPL-3.0) [142]. However, it has a limited library of lattice topology with no interface to define more topology in a simple way. Similar to Intralattice, heterogenous lattice structures are not supported. There has been research on using both Intralattice and Crystallon for the same project with Intralattice used for topology generation and Crystallon used for nodes generation [143].

RHINOCEROS 3D uses polygonal representation at the core of its GMK which is limited when rendering highly complex structures. The General Lattice Studio (GL Studio) plugin for Rhinoceros attempts to improve the performance of RHINOCEROS 3D by bringing B-rep to a format readable by Rhinoceros GMK [144]. GL Studio allows modeling pseudo-periodical lattice structure which provides a certain degree of heterogeneity to the lattice it is applied to, but the general topology remains the same. This, however, is mostly a

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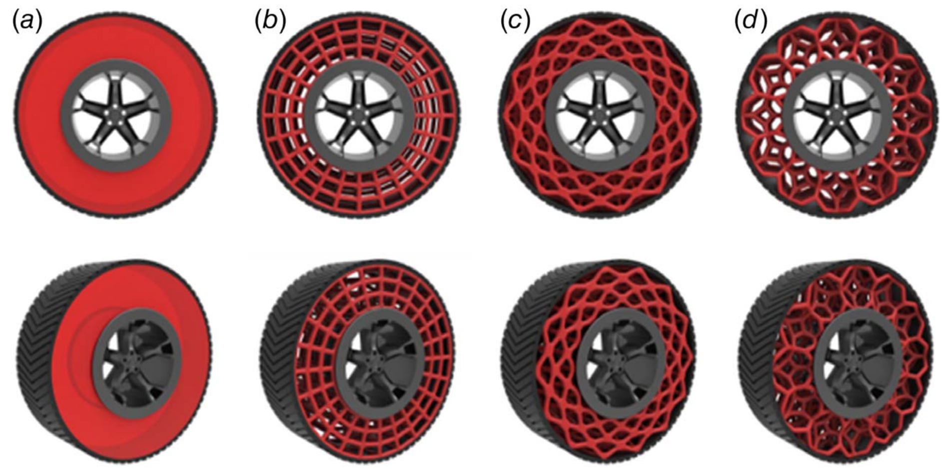

Fig. 20 Tire designs with different lattice topologies, which include (*a*) bare design space, (*b*) grid lattice, (*c*) X lattice, and (*d*) vintiles lattice [140]

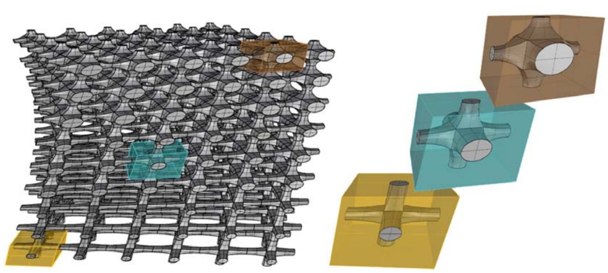

Fig. 21 V-cells of a V-rep model of a heterogeneous lattice [100] (Creative Commons Attribution 4.0 International (CC BY 4.0))

performance improvement tool which still suffers from issues related to B-rep that were described in Sec. 2.

Thus, similar to any other bio-inspired structure, heterogeneous lattice structures lack an appropriate modeling software for their modeling as they require an optimized mesh- or volumetric-based multi-scale geometric modeling approach [30,145].

Beam-based models can be generated via functions by defining them in ANSYS. However, this requires high familiarity with the ANSYS Parametric Design Language which has the syntax similar to it of the FORTRAN programing language [146,147]. Moreover, defining different topologies within a same structure in ANSYS is a complex and not an intuitive process. Beam-based models should also be preprocessed for AM purposes which introduce imperfections in the models, especially in the nodes [148]. Note that volumetric modeling methods can also be applied to heterogeneous lattice structures as they have the same challenges as bio-inspired structures covered in Sec. 2.2 [87].

As seen in Fig. 21, V-rep modeling mentioned in Sec. 2.2 allows defining V-cells with the same topology but with different parameters, both internal (such as the strut diameter) and external (such as the V-cell transformation matrix). The topology itself must be properly defined, as well as all the parameters that are supposed to change throughout the whole structure [100]. This can prove difficult for many non-strut-based topologies such as triply periodic minimal surfaces. The main bottleneck of geometric modeling of complex shapes—significant computational requirement—remains [149]. Moreover, the topology of the same V-rep model should remain the same and a union of several V-rep models is required for a result with varying topology, while there is no guarantee that two models would fit well one into another.

Another geometric modeling technique used in design involves application of F-rep which allows modeling of not only the boundary ($F(X)=0$) but the interior as well ($F(X)\leq 0$). The majority of F-rep methods are incompatible with other modeling formats and cannot store topology information which makes it nontrivial to produce the designed part with AM techniques [150]. However, some F-rep models, such as skeleton-based implicit surfaces, are fully capable of encoding the topology of the solid [151]. Therefore, it is worth investigating whether such approaches could prove beneficial for geometric modeling of complex geometries.

Defining a heterogeneous lattice structure with different topologies is possible through function-based methods by defining areas corresponding to these topologies and assigning weights to the areas, as seen in Figs. 22(*a*) and 22(*b*), or by defining grading functions which change unit cell properties across the design space, as seen in Fig. 22(*c*) [152,153]. However, there are limitations to this method. Even with weights assigned to every topology, some unit cells appear to be disconnected from each other, which negatively impacts the whole model [154]. Designing such lattice structures is not trivial and challenging for a regular user as this involves defining complex function-based rules. Moreover, it is not clear how to design a hierarchical lattice structure with such an approach, as different functions must be defined at each level [155].

Nevertheless, having a function to generate a lattice structure has its own advantages. For instance, consider a previously mentioned torus: in F-rep, the only thing needed for its definition is its implicit equation in Cartesian coordinates, i.e.,

$$\left(\sqrt{x^2+y^2}-R\right)^2+z^2=r^2 \tag{3}$$

where (x, y, z) are the Cartesian coordinates. For another, more complex example, consider a gyroid lattice set by Eq. (3) [156]. The gyroid is considered as an example in this work due to its occurrence in nature in, for example, butterfly wings and a liquid-crystalline lipid mesophase [157,158].

$$\sin(x)\cos(y)+\sin(y)\cos(z)+\sin(z)\cos(x)=0 \tag{4}$$

In HyperFun—an F-rep programing language [159]—only a few lines of code are required to define and model the whole lattice:

```
my_model(x[3], a[1])
{
  my_model = sin(x[1]) * cos(x[2]) + sin(x[2]) *
cos(x[3]) + sin(x[3]) * cos(x[1]);
}
```

Here my_model is set to $F(x)$ which defines the boundary surface $F(x)=0$ in 3D Cartesian coordinates. The resulting surface is illustrated in Fig. 23(*a*).

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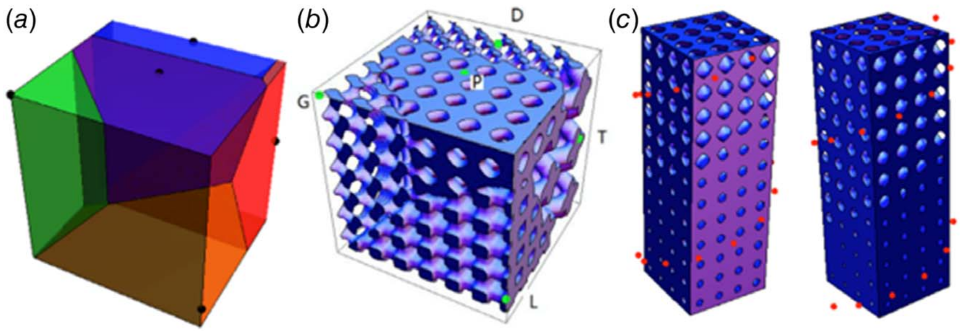

Fig. 22 A heterogeneous lattice can be defined as defining (*a*) weighted areas corresponding to every topology and (*b*) applying actual topologies to the regions, and/or (*c*) changes in unit cell parameters by function-based rules [152] (Permission to reprint from Elsevier © 2015)

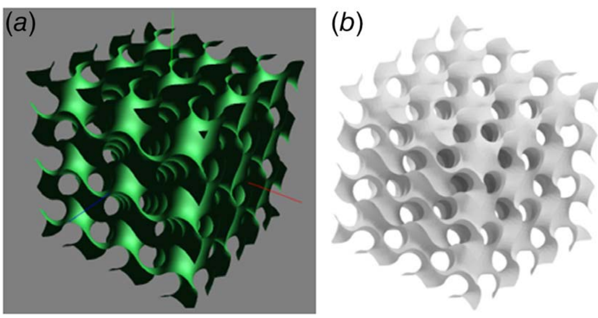

Fig. 23 A gyroid lattice defined by Eq. (4): (*a*) generated by HyperFun language and (*b*) converted to the STL file format, rendered, and ready to be printed

The LSM method described previously in Sec. 2.2 is also applicable to F-rep, as the design space can be sliced into layers and for each layer there can be a 2D function $F(X) \leq 0$ that sets the interior of the solid body for this particular layer [160]. This approach requires defining a step function $S(X)$ that controls the discretization of layers. For example, consider the previously mentioned torus. In this case, for a chosen layer $S_i \subset S$, one would need to define two circles

$$x^2 + y^2 = (R - S(z))^2 \quad \text{and} \quad x^2 + y^2 = (R - S(z))^2 \tag{5}$$

where $S(z)$ serves as the step function and is defined as

$$S(z) = \sqrt{r^2 - z^2} \tag{6}$$

Ideally, $S(X)$ is preferred to be a continuous function, but in LSM it is considered discrete. However, for more complicated structures such as lattice structures, finding a single equation for $F(X)$ is already challenging, and a continuous representation of $S(X)$ is often replaced by a discrete one [161,162]. This introduces unnecessary distortion to the model as the model defined in each layer is a continuous and not a discrete function.

Defining a common homogeneous lattice structure requires to define loops to iterate struts and nodes in all three coordinate directions. However, in heterogeneous case, topologies and rules need to be defined as well, which adds complexity and is not intuitive for a designer. This calls for an intuitive and user-friendly heterogeneous structure generation software.

Designing bio-inspired heterogeneous structures is even more ambiguous for a designer, as this process requires understanding the processes that form geometric shapes in nature up to the ability to explicitly define these shapes as functions [163]. Recall the example with brachistochrone curve from the introduction in Sec. 1: only in the 16th century, scientists were able to formulate and solve the problem of finding an equation of brachistochrone curve [164]. In the 21st century, there are much more complex examples of bio-inspired functions that are applied to designing complex geometric objects [68,165], and nowadays functional description of shapes encountered in nature is still not a trivial and often a computationally expensive task [74,166,167].

F-rep has another advantage over its alternatives: the functions that are used to model an object can serve as an input to a topology optimization algorithm, thus aiding in finding the optimal parameters for heterogeneous lattice structures [168]. Moreover, it has been identified that topology optimization also requires a novel geometric modeling approach [123].

It is important to make sure that the designed CAD file can be open using any machine. Nowadays, the STEP file format defined by the ISO 10303 standard is one of the most popular ones as it can be opened with most of the CAD system and can be directly used for manufacturing [169,170]. However, the STL file format defined by is dominating the AM market as STL files are commonly used as direct inputs for 3D printers [171]. The gyroid lattice in Fig. 23(*a*) can be exported into a STL file format which can be directly used in AM as seen in Fig. 23(*b*).

3.2 Multi-Scale Modeling of Heterogeneous Lattice Structures.

Similarly to other bio-inspired structures, it is required to consider not only the whole heterogeneous lattice structure in its macro-scale but also each joint and strut of its lattice as they form its meso-scale [117,138].

Applying LoD to lattice structures has its own difficulties. One can reduce or increase model complexity by decreasing or increasing, respectively, the amount of polygons needed for its rendering [108]. But when the lattice structure is simplified, i.e., its LoD gets lower, it becomes completely homogenized at some point [172], and when the LoD of a lattice is increase, its size becomes barely manageable, often reaching gigabytes in size, especially in case of heterogeneous lattices [138]. Thus, geometric modeling, design and transferring such structures, becomes slow, making the whole process slow.

There are no clear boundaries between LoDs in models that are not generated by explicit modeling techniques. In lattice structures, the only boundary that can be easily associated with a LoD is when a hierarchical lattice structure is considered—essentially, the hierarchy can be associated with a LoDs as illustrated in Fig. 24. Note that hierarchical lattices are common in nature, e.g., the bamboo structure is hierarchical and has inspired producing similar lattice structures using AM [155].

Topology optimization algorithms often provide unique solutions to the design of multi-scale structures [122,174]. The resulting structures can be heavily heterogeneous geometrically, but the question of designing structures of the same complexity or at least modifying the result of topology optimization is still open.

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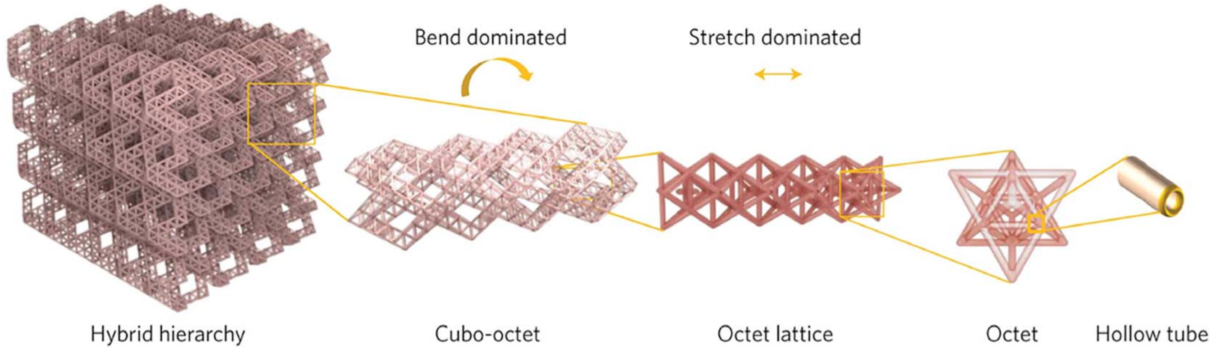

Fig. 24 An example of a hierarchical lattice structure. Note that every next tier in the hierarchy can be associated with a higher level of detail [173] (Permission to reprint from Springer Nature © 2016)

It has been found that a hybrid geometric modeling method based on combining voxel representation and F-rep can serve as a way to model and store the topology information of lattice structures including multi-scale ones [121]. However, it also inherits the high computation time spent on voxel modeling and is not yet fit for direct fabrication.

4 A Proposed Multi-scale Geometric Modeling Framework for Bio-Inspired Complex Structures

Summarizing the above analysis of various literature sources, there is a lack of a geometric modeling method (or tool that would support that method) that suits for geometric modeling of complex geometry and shapes such as heterogeneous lattice structures and other bio-inspired complex structures. The issues with multi-scale modeling in parametric modeling of complex geometry have been identified and covered.

F-rep methods covered in this work allow modeling of homogeneous lattice structures with reasonable ease—a simple lattice structure can be modeled as a self-repeating pattern of unit cells made of cylinders. However, there is still a challenge of defining non-explicit heterogeneous lattices and other bio-inspired complex structures, as their modeling requires to have geometric functions corresponding to them defined first. Functions allow modeling of lattice structures with ease. For example, consider a BCC unit cell sketched in Fig. 25. This unit cell can be described as follows:

$$\left[\begin{array}{ll} x=0, & y=0 \\ x=0, & y=b \\ x=a, & y=0 \\ x=a, & y=b \\ y=0, & z=0 \\ y=0, & z=c \\ y=b, & z=0 \\ y=b, & z=c \\ x=0, & z=0 \\ x=0, & z=c \\ x=a, & z=0 \\ x=a, & z=c \\ \multicolumn{2}{c}{\frac{x}{a}=\frac{y}{b}=\frac{z}{c}} \\ \multicolumn{2}{c}{\frac{x-a}{-a}=\frac{y}{b}=\frac{z}{c}} \\ \multicolumn{2}{c}{\frac{x}{a}=\frac{y}{b}=\frac{z-c}{-c}} \\ \multicolumn{2}{c}{\frac{x-a}{-a}=\frac{y}{b}=\frac{z-c}{-c}} \end{array}\right. \quad \text{for } x \in [0, a],\ \ y \in [0, b],\ \ c \in [0, c] \tag{7}$$

These unit cells could be positioned to the node points of the lattice defined as a point in Cartesian or other coordinates. In a function-

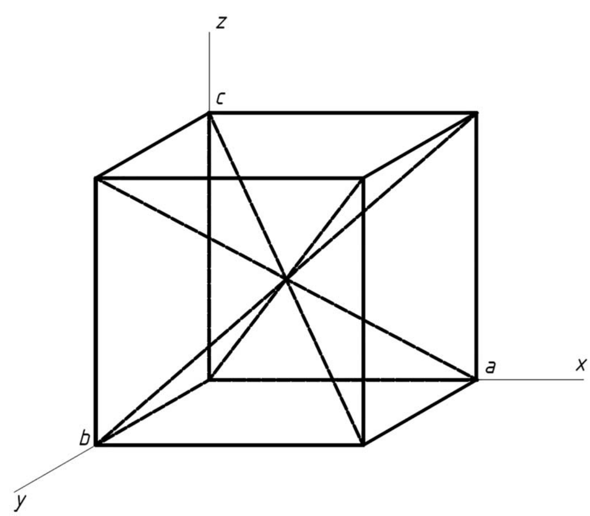

Fig. 25 A BCC unit cell described by Eq. (7)

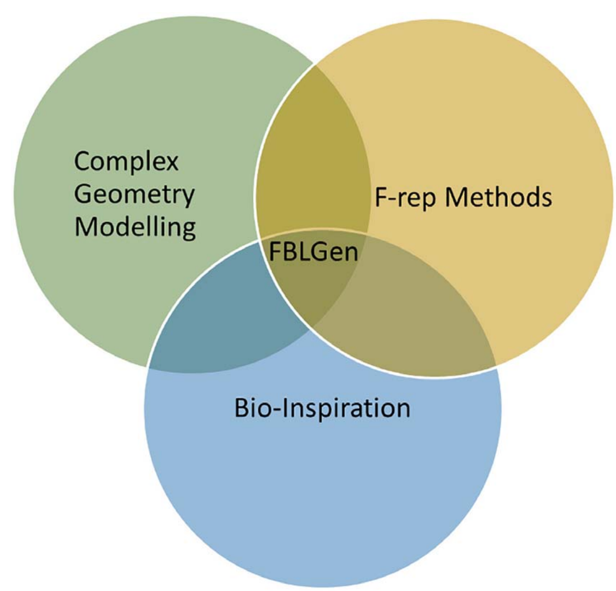

Fig. 26 The ability to model complex geometry using F-rep methods supported by bio-inspired algorithms can result in the FBLGen software tool

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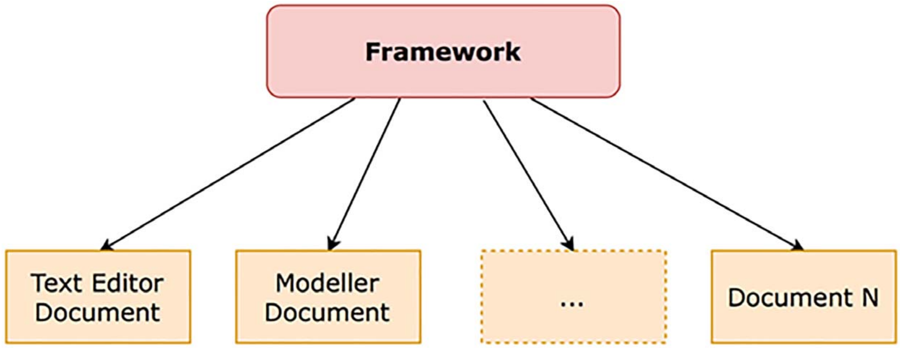

Fig. 27 Top level of the FBLGen architecture

based approach, the lines can easily be replaced by a more complicated equation such as the sine function for example. The nodes of the lattice can also be functionally defined with the most trivial one—a sphere—having a well-known equation. Moreover, as the lines can be replaced with volumetric cylinders, i.e., can form volumetric struts of the lattice, the thickness of these cylinders can be set varying across the whole structure as well as the unit cell bounding parameters a, b, and c. This would also open a possibility to define a lattice inside of another lattice, thus ensuring multi-scale modeling. The user input should be customizable and allow inserting various parameters such as described above.

There is still a concern regarding defining bio-inspired structures with F-rep. A large variety of shapes appear in nature with most of them being non-trivially defined mathematically such as the brachistochrone curve in Fig. 1(*c*). This can be solved by using polygonal or any other interpolation of functions in order to achieve a certain geometry close to bio-inspired. Note that normally there is no need to completely copy nature, as geometric shapes in nature are affected by mutations and other perturbations, making them further from the nature-intended form [175]. There is also a concern of some evolutional developments being useless for the target species. For example, the appendix was notoriously known for bringing unnecessary health issues in human body. Even though the appendix has proved to be useful in a recent research [176], there are still more unnecessary rudimental organs in human body such as the tail bone, the third eyelid, wisdom teeth, etc. Therefore, motivation for choosing the bio-inspired design over the conventional one should be always well justified.

Moreover, there is a potential in application of bio-inspired algorithms and methods to geometry discretization. Combining and integrating together several biological rules related to cell division process is described in Sec. 2.1, such as the bio-inspired geometric

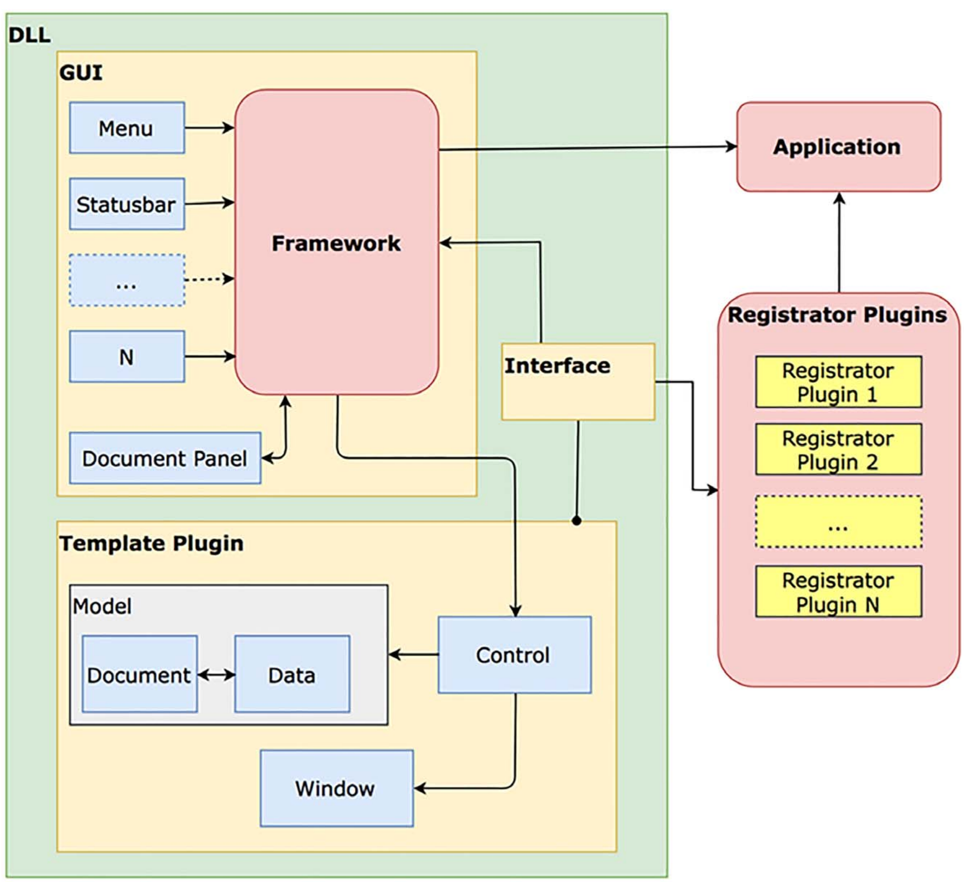

Fig. 28 Low level of the FBLGen architecture

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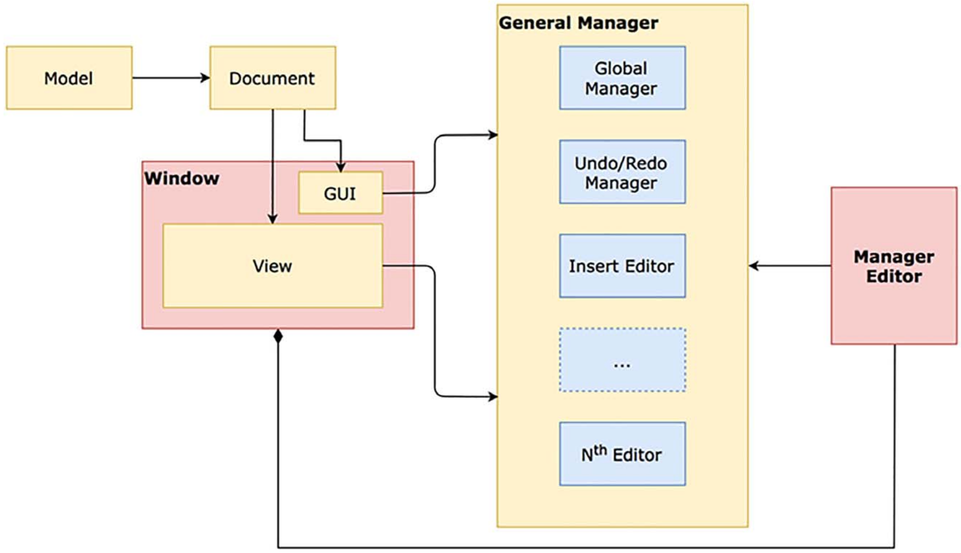

Fig. 29 The diagram representation of the Manager Editor linkage with the window in the FBLGen tool

modeling approach based on volumetric cells [40]. However, there could be more other nature-based algorithms that can be applied to geometric modeling, as the variety of bio-inspired algorithms in such areas as machine learning and simulations is immerse.

Thus, the tool that could be able to model complex geometric structures such as bio-inspired structures can be developed using F-rep methods. There is also a potential in bio-inspired algorithms applied to model discretization similarly to cell division processes occurring in nature. Summarizing, the potential functional bio-inspired and Lattice Geometry Generation software tool (which will be referred to as FBLGen further in this work) could be a result of the research in the intersection of these areas as sketched in Fig. 26.

Even though it is sufficient to develop a console-based application for the sake of proving the concept, it is important to eventually ensure that such lattice design software is user-friendly. Thus, it must follow a specific architecture. The goal of an architecture is to identify the requirements that affect the structure of the application [177]. In other words, an architecture bridges the gap between heterogeneous lattice structure requirements and technical requirements by understanding use-cases, and then finding ways to implement these use-cases in a software. As mentioned in Sec. 2, the design tool shall include a dynamic-link library (DLL) containing a GMK. Moreover, it is a common practice in modern CAD software development to ensure a multi-document interface (MDI) structure, as it helps the designer to operate different kinds of file formats at the same time [178]. In this work, it is assumed that the MDI structure is required for a function-based heterogeneous lattice modeling tool, as the 3D model and the function editor are open in separate documents. Both top level and low level architectures are proposed for the FBLGen tool. In MDI, all documents should be accessed within the single framework on the top level of application architecture as illustrated in Fig. 27.

Figure 28 represents the second—low—level of architecture, where it is important to identify explicit links inside the application. Here, the DLL (which contains a GMK) works together with the graphical user interface (GUI) and the Template Plugin. The GUI is responsible for human–machine interface, while the Template Plugin allows creation of new documents of different types. The

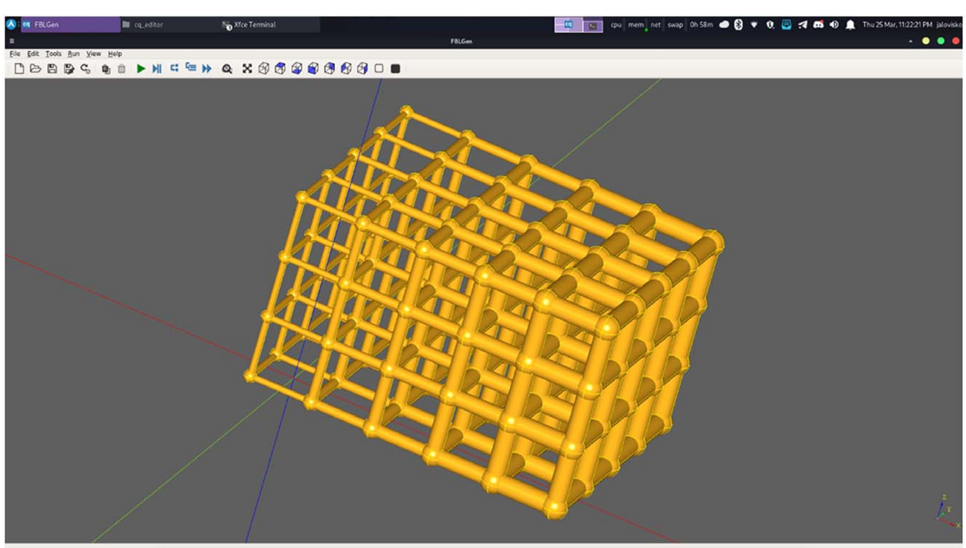

Fig. 30 The prototype of the FBLGen tool used to model a lattice structure with varying strut diameters

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GUI consists of the Framework with various tools such as menu, statusbar, toolbar, etc. The Template Plugin is controlled by the Framework and stores and changes data within the document. It also interacts back with the Framework through the interface. The interface sends signals to the Registrator Plugins, which sends signals with the Framework to the application itself.

The diagram provided in Fig. 29 shows the interconnections between the window and the manager. The model, which corresponds to geometric data, may be represented by the document, which could be seen in the window by a user. The user sends signals to the general manager through the Manager Editor to make changes in the document.

It was decided to develop a prototype of such a tool based on CadQuery tool which enables high parametrization of solid models by scripting them with the PYTHON language, while the GMK is Open CASCADE written in C++ [179]. In an example of a heterogeneous lattice in Fig. 30, a custom script was developed taking the following user input: $d_{\min} = 1$ mm—minimum strut diameter in x direction; $d_{\max} = 3$ mm—maximum strut diameter in x direction; $s = 10$ mm—unit cell size; $N = (5, 4, 3)$—number of unit cells in directions x, y, z. The script then uses a linear function $d(x)$ which depends on $d_{\min}$ and $d_{\max}$ and changes the strut diameter in x direction. Note that the diameter of nodes adapts to user input and is larger where the incoming struts are thicker. It is believed that a certain degree of customizability is required in the user interface to allow user-defined topologies and potentially allow more complex bio-inspired structures.

5 Conclusions

This paper discussed challenges and opportunities arising in geometric modeling of bio-inspired structures with complex geometry such as heterogeneous lattice structures. It was found that the most widely used geometric modeling methods such as polygonal and voxel-based are significantly challenged to support the amount of details that the design freedom of AM provides. Geometric objects essentially become datasets full of entries such as mesh/voxel coordinates and shapes which are extremely large to process. It was identified in this review that either these methods would need to be significantly modified or other methods should be used for highly geometrically complex shapes.

It is found that there is no efficient tool developed to support geometric modeling of such structures. The issues with current geometric modeling methods are identified as related to inability to define different topologies in these methods, inability to correctly model and define the boundaries between topologies, inability to support multi-scale modeling, and general computation efficiency of these methods.

A novel multi-scale geometric modeling framework was proposed to attempt solving these issues in a single software for geometric modeling of complicated bio-inspired structures to be produced by AM.

The future research will be focused, first of all, on supporting geometric modeling of complex 3D structures such as bio-inspired structures and heterogeneous lattice structures. The possibility of having a bio-inspired geometric modeling algorithm to render bio-inspired structures should be investigated further. The mathematical functions that define geometry in nature are not easily found and depend on properties and characteristics of the living organism and its environment. More research is required to develop a method for defining such functions. The software prototype of the proposed FBLGen tool is planned to be developed further to serve as a minimal viable product for the sake of proving the concept. The concept shall be considered proved in case the bio-inspired modeling method can perform better in terms of quantitative and qualitative results compared to alternatives when applied to real and complicated bio-inspired geometric structures such as heterogeneous lattice structures.

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Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

The authors attest that all data for this study are included in the paper.

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